

## COCI 2017/2018

Round #4, December 16th, 2017

### Tasks

Task	Time limit	Memory limit	Score
<b>Rasvjeta</b>	1 s	64 MB	50
<b>Izbori</b>	3 s	64 MB	80
<b>Automobil</b>	1 s	64 MB	100
<b>Vođe</b>	3 s	512 MB	120
<b>Krov</b>	1.5 s	128 MB	140
<b>Ceste</b>	2.5 s	128 MB	160
<b>Total</b>			650

It is Advent season. There are  $M$  street lights in a street  $N$  metres long (the meters of the street are denoted with numbers from 1 to  $N$ ). Each of the lights lights up the meter of the street it's located in and  $K$  meters to the left and to the right of that location. In other words, if the light is located at meter  $X$ , it lights up all metres of the street from  $X - K$  to  $X + K$ , inclusively. Of course, it is possible for a meter of the street to be lit up by multiple street lights. All lights have distinct locations.

The problem is that there is a possibility that the lights don't light up all  $N$  metres of the street. It is your task to determine the minimal amount of additional lights needed to be put up (at position from 1 to  $N$ ) so that the entire street is lit up.

### INPUT

The first line of input contains the number  $N$  ( $1 \leq N \leq 1000$ ).

The second line of input contains the number  $M$  ( $1 \leq M \leq N$ ).

The third line contains the number  $K$  ( $0 \leq K \leq N$ ).

Each of the following  $M$  lines contains a number. The numbers are sorted in ascending order and represent the positions of each of the  $M$  street lights.

The positions will be distinct and from the interval  $[1, N]$ .

### OUTPUT

You must output the required number from the task.

### SAMPLE TESTS

<b>input</b>	<b>input</b>	<b>input</b>
5	26	13
2	3	2
2	3	10
1	3	1
5	19	2
	26	
<b>output</b>	<b>output</b>	<b>output</b>
0	2	1

#### Clarification of the first test case:

It's not necessary to add lights to the street, since all  $N$  meters are already lit up.

#### Clarification of the third test case:

It is necessary to add one lamp, for example at location 13.

In a land with developed democracy far, far away, presidential elections for the football association are taking place. This land consists of  $N$  counties, and each county has its own football association. There are  $M$  presidential candidates labeled with  $1, 2, \dots, M$ . Each of the football associations will select exactly one candidate to cast their vote for. The winner of the election is the candidate with the most votes. If multiple candidates get the most amount of votes, the winner is the one with the smallest label.

During the election campaign, candidates visited the counties and tried to gain their sympathies. After having met all the candidates, each county's football association determined the order in which they would cast their vote for each candidate.

For example, let's assume that there are four candidates in the election and that one county's order is  $2, 1, 4, 3$ . This means that, unless they revoke their candidacy, the candidate with label  $2$  will get the county's vote. If candidate  $2$  revokes their candidacy, and candidate  $1$  is still in the race, then they will get the vote, and so on.

Zdravko is a passionate football fan, and also a close friend of candidate with label  $K$ . He wants to know which candidate will win if neither of the candidates revokes their candidacy.

He also wants to know what is the minimal number of candidates he must persuade to revoke their candidacy in order for his friend, candidate  $K$ , to become the president of the football association.

Zdravko is currently dealing with other problems, so he is hoping that you will answer these questions.

### **INPUT**

The first line of input contains the numbers  $N$  ( $1 \leq N \leq 100$ ),  $M$  ( $1 \leq M \leq 15$ ) and  $K$  ( $1 \leq K \leq M$ ) from the task.

Each of the following  $N$  lines contains the orders given by the counties' football associations, i.e. a permutation of the first  $M$  natural numbers.

### **OUTPUT**

You must output the answers to the questions from the task, each in its own line.

### **SCORING**

The output must consist of two non-empty lines, each containing a single integer. The correct answer to each of the questions is worth 50% of points for that test case.

**SAMPLE TESTS**

**input**

3 4 1  
3 4 1 2  
4 2 3 1  
3 4 2 1

**output**

3  
3

**input**

4 1 1  
1  
1  
1  
1

**output**

1  
0

**input**

4 4 4  
2 3 1 4  
2 3 1 4  
1 3 2 4  
4 3 2 1

**output**

2  
3

**Clarification of the first test case:**

The land where the elections are being held consists of 3 counties, and there are 4 candidates for the president of the association. If neither of the candidates revoke their candidacy, candidate 3 will win the elections with two votes. Candidate 1 will only win if all the other candidates revoke their candidacy.

**Clarification of the second test case:**

There is only one candidate, Zdravko's friend, so they will surely win.

Mirko has found a matrix with  $N$  rows and  $M$  columns at the back seat of his car. The first row of the matrix consists of numbers  $1, 2, \dots, M$ , the second row of numbers  $M+1, M+2, \dots, 2 \cdot M$  and so on until the  $N^{\text{th}}$  row which consists of numbers  $(N-1) \cdot M + 1, (N-1) \cdot M + 2, \dots, N \cdot M$ , respectively.

For example, for  $N = 3$  and  $M = 4$ :

1	2	3	4
5	6	7	8
9	10	11	12

Such matrix wasn't interesting enough to him, so he chose a row or a column  $K$  times and multiplied its values with a non-negative integer.

Naturally, now he wants to know the sum of all the values from the matrix. Since this sum can be very large, Mirko will be satisfied with the value modulo  $10^9 + 7$ . Help Mirko answer this question.

### INPUT

The first line of input contains the numbers  $N$  ( $1 \leq N \leq 1\,000\,000$ ),  $M$  ( $1 \leq M \leq 1\,000\,000$ ) and  $K$  ( $1 \leq K \leq 1000$ ) from the task.

Each of the following  $K$  lines describes:

- Either the multiplication of the  $X^{\text{th}}$  row with  $Y$ , in the form of "R X Y", where "R" represents row multiplication,  $X$  is a positive integer ( $1 \leq X \leq N$ ), and  $Y$  is a non-negative integer ( $0 \leq Y \leq 10^9$ ).
- Or the multiplication of the  $X^{\text{th}}$  column with  $Y$ , in the form of "S X Y", where "S" represents column multiplication,  $X$  is a positive integer ( $1 \leq X \leq M$ ), and  $Y$  is a non-negative integer ( $0 \leq Y \leq 10^9$ ).

### OUTPUT

You must output the sum of the final values from the matrix modulo  $10^9 + 7$ .

### SCORING

In test cases worth a total of 50 points, it will hold  $1 \leq N, M \leq 1000$ .

**SAMPLE TESTS**

**input**

3 4 4  
R 2 4  
S 4 1  
R 3 2  
R 2 0

**output**

94

**input**

3 1 1  
S 1 4

**output**

24

**input**

2 4 4  
S 2 0  
S 2 3  
R 1 5  
S 1 3

**output**

80

**Clarification of the first test case:**

After multiplying the second row with 4, the fourth column with 1, the third row with 2, and again the second row with 0, the final matrix looks like this:

1	2	3	4
0	0	0	0
18	20	22	24

The sum of the elements from the final matrix is  $1 + 2 + 3 + 4 + 0 + 0 + 0 + 0 + 18 + 20 + 22 + 24 = 94$ .

As we all very well know, goats and sheep have been fighting for years about the fields they're grazing. After many fierce fights, the goat leader and the sheep leader decided to meet to try to find a peaceful solution to their problem. After many hours of discussion, they agreed that they will play a game for each field and that the winner will get to graze that field.

The game is played such that a total of  $N$  animals (that can be goats or sheep) form a circle (the exact order of goats and sheep is an agreement between their leaders). After animal  $i$  ( $1 \leq i \leq N-1$ ), the game is continued by animal  $i+1$ , and after animal  $N$ , the game is continued by animal 1. The animal starting the game can say any positive integer from the interval  $[1, K]$ , but only if that number is not greater than  $M$ . If the animal that started the game said the number  $j$ , then the next animal can say a number in interval  $[j+1, j+K]$ , but only if that number is not greater than  $M$ . In other words, each animal can say a number that is greater by minimally 1, and maximally by  $K$  than the number said by the animal before, but only if the new number is not greater than  $M$ . If an animal must say number  $M$ , its team (goats or sheep) loses.

If both the goats and the sheep are playing optimally, for each  $i$  ( $1 \leq i \leq N$ ), determine who will win the field if the game is started by the  $i^{\text{th}}$  animal.

### INPUT

The first line of input contains  $N$ ,  $M$  and  $K$  ( $1 \leq N, M, K \leq 5000$ ), the numbers from the task. The following line contains  $N$  numbers, 0 if the  $i^{\text{th}}$  animal is a sheep, and 1 if it's a goat.

### OUTPUT

Output  $N$  space-separated numbers. For each animal  $i$  ( $1 \leq i \leq N$ ) output 0 if the sheep will win the field, and 1 if the goats will win, if the  $i^{\text{th}}$  animal is starting the game.

### SCORING

In test cases worth a total of 60% of points, it will hold  $1 \leq N, M, K \leq 500$ .

### SAMPLE TESTS

<b>input</b>	<b>input</b>	<b>input</b>
2 9 2	6 499 5	10 100 10
0 1	1 0 0 1 1 0	0 0 0 1 1 1 1 0 1 1
<b>output</b>	<b>output</b>	<b>output</b>
0 1	0 1 1 1 1 0	1 1 1 1 1 1 1 1 1 1

**Clarification of the first test case:**

When a sheep is playing first, it can play like this:

The sheep starts with number 2, after which the goat can say 3 or 4. In both cases, the sheep can say 5, after which the goat can say either 6 or 7. In both cases, the sheep can say 8, after which the goat doesn't have any other choice but 9 and thus losing the game and the field.

You are given a histogram consisting of  $N$  columns of heights  $X_1, X_2, \dots, X_N$ , respectively. The histogram needs to be transformed into a roof using a series of operations. A roof is a histogram that has the following properties:

- A single column is called the top of the roof. Let it be the column at position  $i$ .
- The height of the column at position  $j$  ( $1 \leq j \leq N$ ) is  $h_j = h_i - |i - j|$ .
- All heights  $h_j$  are positive integers.

An operation can be increasing or decreasing the heights of a column of the histogram by 1. It is your task to determine the minimal number of operations needed in order to transform the given histogram into a roof.

### INPUT

The first line of input contains the number  $N$  ( $1 \leq N \leq 10^5$ ), the number of columns in the histogram.

The following line contains  $N$  numbers  $X_i$  ( $1 \leq X_i \leq 10^9$ ), the initial column heights.

### OUTPUT

You must output the minimal number of operations from the task.

### SCORING

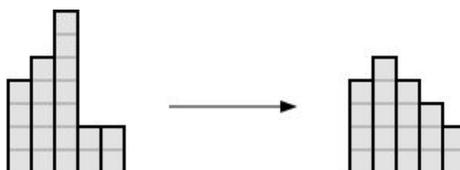
In test cases worth 60% of total points, it will hold  $N \leq 5000$ .

### SAMPLE TESTS

<b>input</b>	<b>input</b>	<b>input</b>
4	5	6
1 1 2 3	4 5 7 2 2	4 5 6 5 4 3
<b>output</b>	<b>output</b>	<b>output</b>
3	4	0

**Clarification of the first test case:** By increasing the height of the second, third, and fourth column, we created a roof where the fourth column is the top of the roof.

**Clarification of the second test case:** By decreasing the height of the third column three times, and increasing the height of the fourth column, we transformed the histogram into a roof. The example is illustrated below.



There's a country with  $N$  cities and  $M$  bidirectional roads. Driving on road  $i$  takes  $T_i$  minutes, and costs  $C_i$  kunas (Croatian currency).

To make the arrival to the holiday destination as pleasant as possible, you want to make it as fast and as cheap as possible. More specifically, you are in city 1 and want to minimize the product of total money spent and total time spent (overall, with all roads you drove on) in getting to a city from city 1. For each city (except city 1), output the required minimal product or -1 if city 1 and that city aren't connected.

### INPUT

The first line of input contains numbers  $N$  ( $1 \leq N \leq 2000$ ), the number of cities, and  $M$  ( $1 \leq M \leq 2000$ ), the number of roads.

Each of the following  $M$  lines contains four numbers,  $A_i, B_i, T_i, C_i$  ( $1 \leq A_i, B_i \leq N, 1 \leq T_i, C_i \leq 2000$ ) that denote there is a road connecting cities  $A_i$  and  $B_i$ , that it takes  $T_i$  minutes to drive on it, and it costs  $C_i$  kunas.

It is possible that multiple roads exist between two cities, but there will never be a road that connects a city with itself.

### OUTPUT

You must output  $N - 1$  lines. In the  $i^{\text{th}}$  line, output the required minimal product in order to get to city  $(i + 1)$ , or -1 if cities 1 and  $(i + 1)$  aren't connected.

### SCORING

In test cases worth 40% of total points, it will hold  $1 \leq N, M, T_i, C_i \leq 100$ .

### SAMPLE TESTS

**input**

```
4 4
1 2 2 4
3 4 4 1
4 2 1 1
1 3 3 1
```

**output**

```
8
3
14
```

**input**

```
4 5
1 2 1 7
3 1 3 2
2 4 5 2
2 3 1 1
2 4 7 1
```

**output**

```
7
6
44
```

**input**

```
3 2
1 2 2 5
2 1 3 3
```

**output**

```
9
-1
```

**Clarification of the second test case:**

In order to get to city 2, you need to drive on road 1, for that it takes 1 minute and 7 kunas, so the required product is 7.

In order to get to city 3, you need to drive on road 2, for that it takes 3 minutes and 2 kunas, so the required product is 6.

In order to get to city 4, you need to drive on roads 2, 4, 5, in that order, and for that it takes a total of 11 minutes and 4 kunas, so the required product is 44.