

KTH Challenge 2017 Solutions

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Problem

Minimum number of days to print n statues with a 3D printer that can replicate itself.

Solution

- 1 While not enough printers to print all statues:
 - ▶ Print more printers
- 2 Print all statues

XX submissions, YY correct, first at 0:03.

Problem

Find smallest $h \geq h_0$ such that number of cards in triangular card house of height h is multiple of 4.

Solution

- 1 Number of cards is $\frac{3h^2+h}{2}$:
 - ▶ $1 + 2 + \dots + h - 1 = \frac{h(h-1)}{2}$ horizontal cards
 - ▶ $2 + 4 + 6 + \dots + 2h = h(h+1)$ diagonal cards
- 2 Evaluate formula for $h_0, h_0 + 1, h_0 + 2$ etc until we get a multiple of 4 and hope that it works without knowing why
- 3 Or note that we want $3h^2 + h \equiv 0 \pmod{8}$
 - ▶ Solutions are $h \equiv_8 0$ and $h \equiv_8 5$
 - ▶ Set $h = h_0 + \min(-h_0 \bmod 8, (5 - h_0) \bmod 8)$

XX submissions, YY correct, first at H:MM:SS.

E – Global Warming

Problem

Minimize environmental impact when arranging students into groups of two

Insights

- ▶ Minimum cost perfect matching, non-bipartite graph
- ▶ Graph is undirected and transitive: disjoint union of cliques
- ▶ Since $|E| \leq 250$, each clique has size at most 22

Solution

- ▶ Find the connected components (the cliques)
- ▶ “impossible” if any clique has odd no. of vertices
- ▶ $O(n2^n)$ dynamic programming on each clique

XX submissions, YY correct, first at H:MM:SS.

Problem

Determine if it possible to arrange two sets of cards such that no two cards have the same suit, or the last card of one part is the same suit as the other part but lower rank.

Insights

- ▶ Need at least 26 cards
- ▶ Ranks matter only when exactly 26 cards
- ▶ Then, there must be a suit where we have a higher card than the opponent

Insights

- ▶ This is also sufficient
- ▶ Hall's Marriage Theorem: If we have A, B, C, D cards of the four suits and the opponent has $13 - A, 13 - B, 13 - C, 13 - D$, then

$$13 - A \leq B + C + D...$$

is sufficient for a matching, which is clearly true.

Short solution

```
def solve(n):
    if n != 26: return n > 26
    has = [[0] for _ in range(4)]
    for _ in range(n):
        (v, s) = raw_input().split()
        has['CDHS'.find(s)].append(int(v))
    return next((True for i in range(4)
                if max(has[i]) > len(has[i])-1), False)

print ['impossible', 'possible'][solve(int(raw_input()))]
```

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G – Restaurant Bribes

Problem

Maximize money earned by bribing people in social network to give good ratings

Insight

The effect of each bribe is independent of other bribes

Solution

For each bribed person:

- ▶ Compute total influence on their friends
- ▶ Compute which number of stars maximizes money earned

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Problem

Compute sum of weird function $f(x)$ over an interval.

Insights

- ▶ We can transform the problem to only computing the interval $[1, X]$
- ▶ For $[1, 1]$, the answer is 0
- ▶ For $[1, 2X]$, the answer is $X/2 + X - 2 + 2F(X)$
- ▶ For $[1, 2X + 1]$, the answer is $F(2X) + f(2X + 1)$

XX submissions, YY correct, first at H:MM:SS.

Problem

Add minimum undirected edge to make directed graph connected

Solution

- ▶ Compute reachability graph
- ▶ Compute strongly connected components
- ▶ Compute which components are sources or sinks
 - ▶ None: no need for an extra edge
 - ▶ Two: choose minimum edge between components
 - ▶ More: no solution

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I – Lights Out

Problem

Find shortest route in building such that always possible to turn off all lights

Solution

- ▶ A switch is useful if it can turn out more subsets than the previous switches
- ▶ Equivalent: it is linearly independent in \mathbb{F}_2
- ▶ Task: find shortest path in graph that visits a set of switches of max rank
- ▶ Solution: BFS with state (room, LI matrix)

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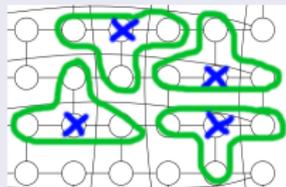
A – Saturn Bees

Problem

Decide if dominating set of a hexagonal grid is $mn/4$

Insight

Only two possible patterns



Solution

- ▶ If m and n even, closed formula with pattern 1
- ▶ Otherwise, test if pattern 2 works

XX submissions, YY correct, first at H:MM:SS.

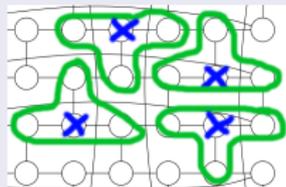
A – Saturn Bees

Problem

Decide if dominating set of a hexagonal grid is $mn/4$

Insight

Only two possible patterns



Solution

- ▶ If m and n even, closed formula with pattern 1
- ▶ Otherwise, test if pattern 2 works
- ▶ Actually, have closed formula too

XX submissions, YY correct, first at H:MM:SS.

This was fun! When is the next contest?

- ▶ We train every two weeks at KTH, check www.csc.kth.se/contest
- ▶ Next training in August/September
- ▶ Nordic Championships in October, North-western Europe qualifier in November
- ▶ New Swedish League starting in August, check codingcup.se
- ▶ Plenty of other online competitions every week
- ▶ Subscribe to our calendar

Guide To Programming Contests

- ▶ <https://lukipuki.github.io/contest-wiki/>
- ▶ Written by Lukáš.
- ▶ The first training program for programming contests.
- ▶ Well received in the contest community.