KTH Challenge 2015 Solutions

April 26, 2015

Jury

- Erik Aas (Paris 7)
- Oskar Werkelin Ahlin (Spotify)
- Per Austrin (KTH)
- Jan Elffers (KTH)
- Simon Klein (KTH)
- Ulf Lundström (Stanford)
- Lukáš Poláček (Spotify/KTH), head of jury
- Marc Vinyals (KTH)

B – Black Friday

Problem

Determine the winner of a dice rolling game where the highest **unique** outcome wins.

Solution

- Divide players into buckets based on their outcomes.
- Find the highest outcome with bucket size 1 (if any).
- Output the player number.

47 submissions, 38 correct, first at 0:03:23.

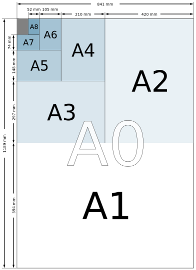
G – A1 Paper

Problem

Find length of tape needed to make an A1 paper.

Solution

- Start with the largest papers.
- Keep track of how many sheets of the current size that must be created.
- This is twice the number needed at previous size minus the number of sheets we have of this size.
- Add the length of tape needed for this to a running total.



71 submissions, 28 correct, first at 1:25:43.

C – Absurdistan Roads III

Problem

Orient n edges among n vertices, so that each vertex has one outgoing edge.

Solution

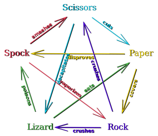
- Orient all leaves towards their neighbors and remove them from the graph.
- Repeat until no leaves are left.
- The rest is just cycles, orient them arbitrarily.
- Can be implemented in linear time.

83 submissions, 17 correct, first at 0:23:08.

F – Spock

Problem

Predict the moves made by a simple pseudo random number generator.



Solution

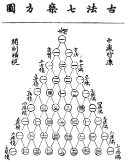
- Keep a list of all $p^3 \approx 2M$ possible states of the RNG
- When making a move:
  pick a possible state at random and output a move which beats the move predicted by that state.
- When getting a move from the computer:
  filter out all states that did not predict that move.
- (Optimization: by making a few arbitrary moves in the beginning, can get much shorter list of initial possible states.)

21 submissions, 6 correct, first at 2:14:22.

H – Odd Binomial Coefficients

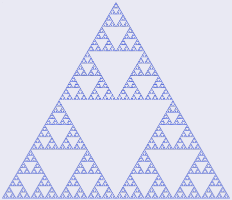
Problem

Count the number of odd binomial coefficients in the first n rows of Pascal's triangle.



Solution

- Print the first n rows of Pascal's triangle modulo 2. Sierpinski triangle:



- Each proper triangle contains 3^k one's, where k is the number of recursion levels.

Solution (continued)

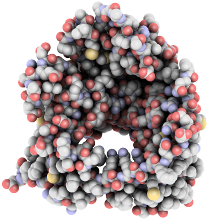
- The first 2^ℓ rows are occupied by a level ℓ triangle, where ℓ is the highest bit in n. Recurse on the bottom $n - 2^\ell$ rows.

27 submissions, 12 correct, first at 0:16:51.

A – Proteins

Problem

Find the minimum number of insertions needed into a string to make N of its 3-letter blocks being "ATG".



Solution

- Keep track of the number of ATG blocks that can be obtained from the last i letters of the original string if inserting j letters.
- A recursive formula can be found.
- Use the fact that substrings "AT", "AG", and "TG" can be turned into "ATG" with one insertion and that "A", "T", and "G" requires two insertions.
- Implement with dynamic programming.

17 submissions, ?? correct, first at 0:23:33.

E – Shibuya Crossing

Problem

Find the size of the maximum clique in a special graph.

Solution

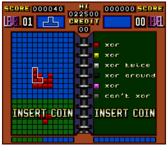
- The input graph is called permutation graph.
- Consider the permutation represented by this graph in one-line notation.
- The largest clique is equal to the longest decreasing subsequence of this sequence.
  - This can be solved efficiently in $O(n \log n)$, though slower solutions would also pass.

15 submissions, ?? correct, first at 1:37:40.

D – Xortris

Problem

Decide if a pattern of black squares is a composition of tetrominoes.



Insight

3×2 or larger boards with even number of black squares are always solvable. **Proof:** You can place a 2×1 piece anywhere with an S and a T. This means you can move one black square one step in any direction until it cancels out with another black square.

Solution

- $1 \times n$: add 1×4 pieces greedily.
- 2×2: check if 2×2 fits.
- otherwise: Check if there is an even number of black squares.

18 submissions, 7 correct, first at 1:20:44.

I – The Addition Game

Problem

Given a, find permutations π and σ such that $a = \pi + \sigma$.

Heuristic solution

- Necessary condition: $a_1 + \cdots + a_n \equiv 0 \pmod{n}$. If this is satisfied, a pair of permutations exist.
- Better formulation: Find a permutation π such that $a - \pi$ is a permutation.
- Heuristic: If $a - \pi$ is not a permutation, some values $k \in \{1, \ldots, n\}$ occur more than once and some values l do not occur in $a - \pi$.
- Make a swap in π to change a k into an l in $a - \pi$. This decreases or keeps constant the number of k's.

Deterministic solution

- Solving the case when $a - \pi$ lacks only two values suffices to solve the whole problem, by changing a step by step from $a = \mathbf{0}$ (for which $\pi_i = i, \sigma_i = -i$ work).
- So $a - \pi$ is missing m_1 and m_2 in positions i_1 and i_2. Find a j such that $a - \pi(i_1 \quad j)$ has value m_1 at position i_1 (we had another possibility, i_2, but we chose i_1).
- Now $a - \pi$ is missing $a_j - \pi_j$ and m_2 in positions j and i_2, and only one possibility (swapping to make $a - \pi$ take value m_2 at i_2) gives us something new.
- Process cannot continue forever, so eventually the missing values are obtained by simply swapping the new i_1 and i_2.
- Published solution in Marshall Hall Jr. "A Combinatorial Problem on Abelian Groups".

10 submissions, ?? correct, first at ?:??:??.

This was fun! When is the next contest?

- We train every two weeks at KTH, check www.csc.kth.se/contest.
- Next training on Wednesday April 29 at 17:15 in Röd.
- Nordic Championships in October, North-western Europe qualifier in November.
- Plenty of other online competitions every week.
- Subscribe to our calendar and RSS feed.

Boot camp May 8 – May 10

- 3 days on Möja in the archipelago.
- Lectures,
  trainings and fun activities.
- By invitation only.
- Also camp for Swedish IOI team.

Photo by The U.S. Army