KTH Challenge 2011

Stockholm, April 9 2011

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Problem A
Reversed Binary Numbers
Problem ID: binary

Yi has moved to Sweden and now goes to school here. The first years of schooling she got in China, and the curricula do not match completely in the two countries. Yi likes mathematics, but now... The teacher explains the algorithm for subtraction on the board, and Yi is bored. Maybe it is possible to perform the same calculations on the numbers corresponding to the reversed binary representations of the numbers on the board? Yi dreams away and starts constructing a program that reverses the binary representation, in her mind. As soon as the lecture ends, she will go home and write it on her computer.

Task

Your task will be to write a program for reversing numbers in binary. For instance, the binary representation of 13 is 1101, and reversing it gives 1011, which corresponds to number 11.

Input

The input contains a single line with an integer N, $1 \leq N \leq 1\,000\,000\,000$.

Output

Output one line with one integer, the number we get by reversing the binary representation of N.

Sample input 1	Sample output 1
`13`	`11`

Sample input 2	Sample output 2
`47`	`61`

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Problem B
Paintball
Problem ID: paintball

Marek and his schoolmates have just finished their studies at the university. They wanted to celebrate it with a game of paintball. After an hour of playing a very strange thing happened – everyone had exactly one bullet left. Marek, being a very curious person, wanted to know whether it's possible that everyone will be hit exactly once provided nobody moves.

Task

You are given a description of the situation during a paintball game when every player has only one bullet. The description of the game consists of pairs of players who can see each other. If a player can see another player, he can fire at him. Your task is to find a target for each player such that everyone will be hit.

Input

The first line of input contains two space separated integers N and M, satisfying $2 \leq N \leq 1\,000$ and $0 \leq M \leq 5\,000$, where N is the number of players. Players are numbered $1, 2, \ldots, N$. M lines follow, each line containing two space separated integers A and B ($1 \leq A < B \leq N$), denoting that players A and B can see each other. Each pair of players appears at most once in the input.

Output

If there is no assignment of targets such that everyone will be hit, output `Impossible`. Otherwise output N lines. The i-th line should contain the number of the target of the i-th player. If there is more than one solution, output any one.

Sample input 1

```
3 3
1 2
2 3
1 3
```

Sample output 1

```
2
3
1
```

Sample input 2

```
3 2
1 2
1 3
```

Sample output 2

```
Impossible
```


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Problem C
Base-2 Palindromes
Problem ID: palindrome

A positive integer N is a *base-b palindrome* if the base-b representation of N is a palindrome, i.e. reads the same way in either direction. For instance, 7 (base 10) is a palindrome in any base greater than or equal to 8. It is also a palindrome in base 2 (111) and 6 (11), but not in 3 (21), 4 (13), 5 (12), or 7 (10). The first four base 2 palindromes (written in base 10) are 1, 3, 5, and 7.

Task

You are supposed to find the M-th base-2 palindrome and output its base 10 representation.

Input

The input is a single line with a single positive integer $M \leq 50\,000$ in base 10.

Output

The output for input M should be a single line with the base 10 representation of the M-th base-2 palindrome.

Sample input 1	Sample output 1
1	1

Sample input 2	Sample output 2
3	5

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Problem D
Kindergarten Excursion
Problem ID: excursion

The kindergarten teachers had finally managed to get all the kids in a line for the walk to the bus station and the weekly excursion. What they hadn't thought of was the fact that today different kids were supposed to go to different excursions. They should walk as one line to the bus station, but to avoid total chaos when they arrive there, all the kids going to the zoo should be in the beginning of the line, the ones going to the lake should be in the middle and the rest, going to the science museum, should be in the end.

Since it takes a lot of time to get all the kids to stand in a line no kid may step out of the line. To get the line organized after excursion group, kids standing next to each other can swap places. Now the kindergarten teachers wonder, if they will have time to do this and still catch their bus.

Task

You will be given a sequence of numbers 0, 1, and 2, denoting the excursion destinations of each kid from first to last in the line. Pairs of adjacent kids can swap positions, and the line should be organized after destination number starting with 0 and ending with 2. What is the minimum number of swaps required to organize the line?

Input

The only line of input contains a string consisting of characters 0, 1 or 2, denoting the destinations of kids. The length of the string will be at most 1 000 000 characters.

Output

Output one line with one integer – the minimum number of swaps needed to get the kids in order.

Sample input 1	Sample output 1
`10210`	`5`

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Problem E
Coast Length
Problem ID: coast

The island municipality of Soteholm is required to write a plan of action for their work with emission of greenhouse gases. They realize that a natural first step is to decide whether they are for or against global warming. For this purpose they have read the IPCC report on climate change and found out that the largest effect on their municipality could be the rising sea level.

The residents of Soteholm value their coast highly and therefore want to maximize its total length. For them to be able to make an informed decision on their position in the issue of global warming, you have to help them find out whether their coastal line will shrink or expand if the sea level rises. From height maps they have figured out what parts of their islands will be covered by water, under the different scenarios described in the IPCC report, but they need your help to calculate the length of the coastal lines.

Task

You will be given a map of Soteholm as an $N \times M$ grid. Each square in the grid has a side length of 1 km and is either water or land. Your goal is to compute the total length of sea coast of all islands. Sea coast is all borders between land and sea, and sea is any water connected to an edge of the map only through water. Two squares are connected if they share an edge. You may assume that the map is surrounded by sea. Lakes and islands in lakes are not contributing to the sea coast.

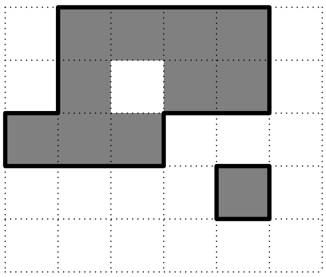

Figure E.1: Gray squares are land and white squares are water. The thick black line is the sea coast. This example corresponds to Sample Input 1.

Input

The first line of the input contains two space separated integers N and M where $1 \leq N, M \leq 1000$. The following N lines each contain a string of length M consisting of only zeros and ones. Zero means water and one means land.

Output

Output one line with one integer, the total length of the coast in km.

Sample input 1	Sample output 1
5 6 011110 010110 111000 000010 000000	20

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Problem F
Icons in the Toolbar
Problem ID: icons

Gunnar is not satisfied with the graphic tools currently available for LaTeX, so he's working on his own graphics editor. For every editor function he has also created an icon which is a square image. These icons will be placed in a toolbar with two rows and N columns. The height of a toolbar row is the same as the size of the biggest icon in the row. Similarly, the width of a column is the same as the size of the biggest icon in the column. The height of the toolbar is the sum of heights of all rows and the width of the toolbar is the sum of widths of all columns. Gunnar now wonders how he should order the icons so that the area of the toolbar is as small as possible.

100	99	98
3	1	2

Figure F.1: The height of this toolbar is $100 + 3$ and the width $100 + 99 + 98$. The total area is $(100 + 3) \cdot (100 + 99 + 98) = 30\,591$. This example corresponds to Sample Input 1.

Task

You will be given a sorted integer sequence $s_1, \ldots, s_{2N}$ denoting that the i-th icon has dimensions $s_i \times s_i$. Your task is to calculate the smallest possible area of a toolbar with 2 rows and N columns.

Input

The first line of the input contains one integer $N, 1 \leq N \leq 1\ 000\ 000$. Each of the next $2N$ lines contains one integer s_i denoting the size of an icon. You can assume that $1\ 000\ 000 \geq s_1 \geq s_2 \geq \ldots \geq s_{2N} \geq 1$.

Output

The output consist of a single line with one integer – the smallest possible area of the toolbar.

Sample input 1	Sample output 1
3 100 99 98 3 2 1	30591

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Problem G
Getting Rid of Coins
Problem ID: coins

When Per was living in New York, he couldn't use his debit card as often as in Sweden. He had to carry cash all the time and his wallet was getting heavy from all the coins he was carrying around. He was once in a candy store buying a few kilos of candy and he wanted to get rid of as many coins as possible but couldn't figure out how to do it without getting any change back.

Task

You will be given a price P that Per was going to pay. You will also be given the number of 1-, 5-, 10- and 25-cent coins that he had in his wallet. He didn't have any notes. Find the maximum number of coins he could have used for paying the price P without getting any change back.

Input

The first line of input contains an integer $P, 1 \leq P \leq 100\,000\,000$, the price Per was going to pay. On the second line there are 4 space separated integers $N_1, N_5, N_{10}, N_{25}; 0 \leq N_1, N_5, N_{10}, N_{25} \leq 100\,000\,000$, the number of 1-, 5-, 10- and 25-cent coins in Per's wallet.

Output

If Per couldn't pay exactly P without getting any change back, output `Impossible` on a single line. Otherwise output the maximum number of coins Per could use to pay the price P.

Sample input 1	Sample output 1
13 3 2 1 1	5

Sample input 2	Sample output 2
13 2 2 1 1	Impossible

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