GCPC 2012

GCPC 2012 Jury

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Battleship

- Solve by Simulation
- Read problem statement carefully
- Ending the game and draw may be tricky cases

BrainfuckVM

- Problem: decide whether a program terminates

BrainfuckVM

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- Solution: simulate program while keeping track of states
- simulate concurrently at normal and at double speed (Floyd/Brent's cycle trick)
- if both simulations have equal state: loop!
- needs clever state comparison to be fast enough

BrainfuckVM

- $\Rightarrow$ simpler solution: just simulate 50 000 000 steps
- if not terminated $\rightarrow$ loop
- simulate another 50 000 000 steps to get loop instructions
- no state comparison needed

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- equation solvable if K and C coprime

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- Possible solution:
  - Minimize both DFAs (see Hopcroft/Ullman)
  - Check for equivalence by for example DFS:
  - Simultaneous DFS on the two minimized automatas numbering the states in preorder
  - Check if the state reached by an input character has same DFS number or is unvisited
  - Check if the final states have the same DFS number
  - Runs in $O(|states|^2 \cdot |\Sigma|)$

Outsourcing

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Outsourcing

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  - Join the automata
  - Union states that are reached by the same inputs by DFS
  - After all, check if there is a union of exactly the two final states
  - Really fast (nearly $O(|states| + |transitions|)$) with a good union-find implementation

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- Solution: Match words which appear as ingredients on the same set of Pizzas
- Use bitmasks to specify for each ingredient the subset of Pizzas on which this ingredient occurs.
- Brute force over all pairs of words and check if their corresponding bitmasks are equal

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- *Unbounded Knapsack*:

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- 0/1-*Knapsack*:

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  J_i is the largest index where the profit is positive.
  Use $0/1$-*knapsack* on those items!

Roller coaster fun

Time complexity:

- Build table: $O(N \cdot J_{max} \cdot T_{max})$
- Query table entries: $O(Q)$

Only one testcase, $J_{max} \leq 32$
$\Rightarrow$ time complexity is sufficient

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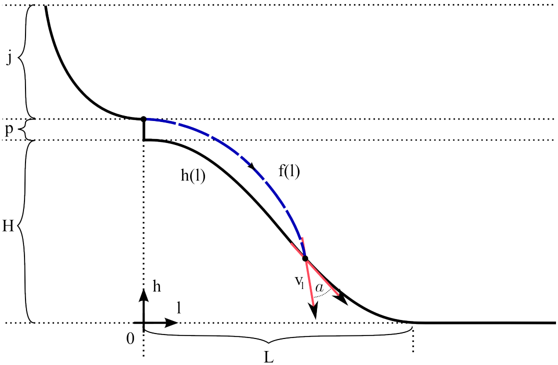
$\rightsquigarrow$ Memory: $O(2 \cdot T_{max}) < 1\text{MB}$

Ski Jumping



- Find l
  1. by solving for $l \rightarrow$ much math, paper and pencil approach, $\mathcal{O}(1)$
  2. by binary search on $l \rightarrow$ easy too implement, $\mathcal{O}(\log N)$

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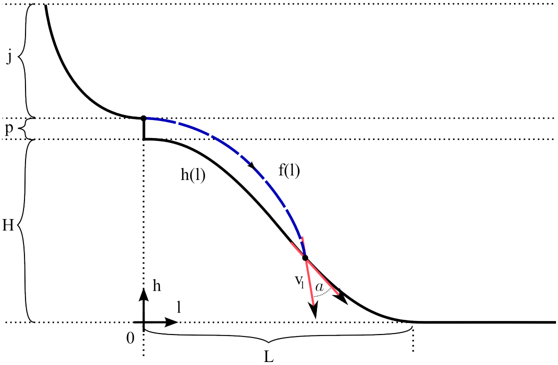

- Get landing speed $|v|$
  - $v_x =$ speed gained in approach, not changed during flight
  - $v_y =$ speed gained during flight (drop since approach)

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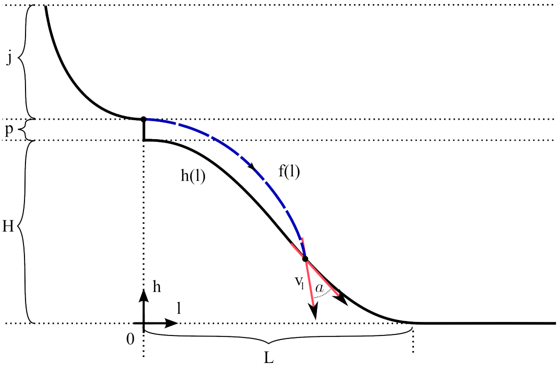

- Get landing angle
  1. First derivatives of f and h yield slopes
  2. Obtain slopes at landing point
  3. Write slopes as vector and apply given equation
  4. Convert rad to degree

Suffix Array RE-construction

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A	S	t	r	i	n	g	W	i	t	h	C	o	n	f	l	i	c	t	s

1 AStringW*Conflicts → 1 AStringW, 12 Conflicts

8 WithC*Licts → 8 WithC, 16 Licts

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    x[keys[i][j]] = i;
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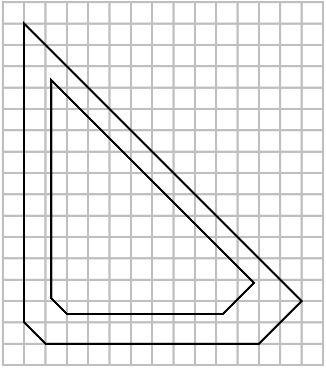
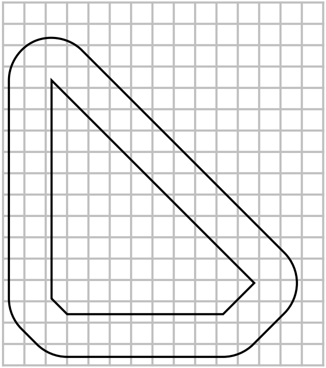
- sort and print

scale with f



Track Smoothing

border with distance r



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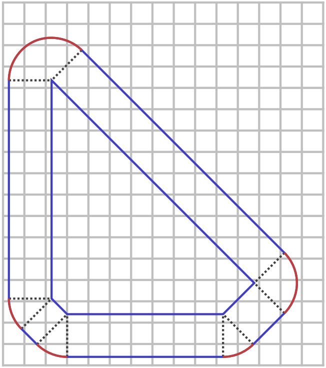
$f \cdot$ track_length



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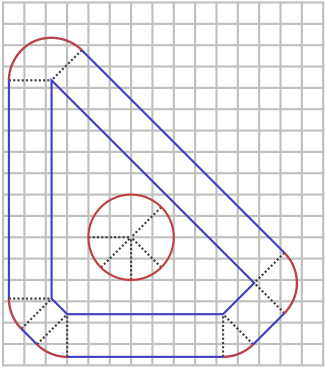
$f \cdot \text{track\_length} + 2r\pi$



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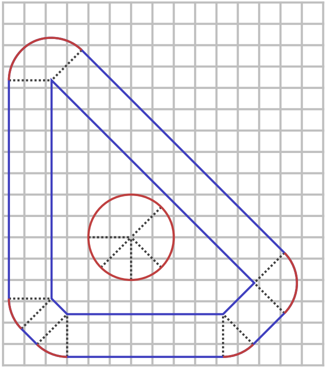
$$f \cdot \text{track\_length} + 2r\pi = \text{track\_length}$$



- $f = \frac{\text{track\_length} - 2r\pi}{\text{track\_length}}$

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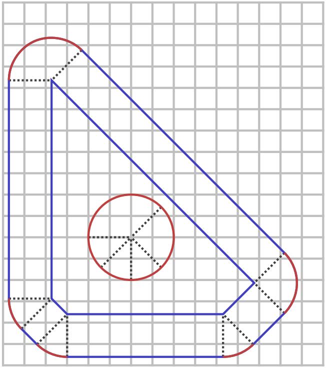

- $f = \frac{\text{track\_length} - 2r\pi}{\text{track\_length}}$
- negative $\Rightarrow$ "Not possible"

Treasure Diving

- Problem: decide how many Treasures a diver can rescue from a cave network using a limited air budget

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- Classical TSP instance, with a minor twist
- The diver does not have to collect all treasures, only maximal number possible
- Two step approach
  - calculate distance table
    (at most 8 Treasures + exit → 9x9 Table)
  - perform backtracking on table, recursing only if air sufficient for the return

Award Ceremony