Day 4: LiJie Chen Contest

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A. Life game

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  - of size up to 50

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  - x_1, x_2, y_1, y_2, c and m
  - You get an award m if submatrix (x_1, x_2, y_1, y_2) is colored in c

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- Earn maximum amount of money

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- Do for $c = 1$ in the same way
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- One can see, that if award criterion is not satisfied, then m-valued edge is intersecting a cut
- This will give up to $50^2 \cdot 50\,000$ edges and $50 + 50\,000$ vertices
  - Pretty much for maxflow algorithms

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Optimizing

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  - Add 4 edges from 4 submatrices of power-of-2 sizes to an award as in 2D sparse table
  - We don't care if they overlap

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  - Add 4 edges from 4 submatrices of power-of-2 sizes to an award as in 2D sparse table
  - We don't care if they overlap
- Add $+\infty$ edges between power-of-2 size submatrices
  - From big one to two times smaller ones

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- As it turns out, this optimization helps to get passed TL
  - Fast algorithms like Dinitz or Preflow-push algorithms help
- Here we have about $2 \cdot 50^2 \cdot \log^2 50$ edges for sparse table
- $5 \cdot 50\,000$ edges for edges between awards and submatrices
- And $2 \cdot 50^2 \cdot \log^2 50 + 50\,000$ vertices

B. Reincarnation

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- You are also given queries
  - Given L and R
  - Find number of different substrings in $s(L, R)$

B. Reincarnation

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- Substring $s(i, j)$ is inside all (x, y) for $x \leqslant i$ and $j \leqslant y$
  - For all $i \leqslant j$ add 1 to all $f_{x,y}$ such that $x \leqslant i$ and $j \leqslant y$

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  - $t = s(i_1, j_1) = s(i_2, j_2) = \ldots = s(i_k, j_k)$
    - $i_1 < i_2 < \cdots < i_k$
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    - for $1 \leqslant e < k$

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  - Subtract 1 of all $f_{x,y}$ such that $x \leqslant i_e$ and $j_{e+1} \leqslant y$
    - for $1 \leqslant e < k$
- Then just answer all queries outputting $f_{L,R}$

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- So it's $w-(w-1)=+1$ for each substring that is inside

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- And 2D Partial sums will help adding $f_{x,y}$ for $x \leqslant i$ and $j \leqslant y$

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- Partial sums will help iterating over g
- And 2D Partial sums will help adding $f_{x,y}$ for $x \leqslant i$ and $j \leqslant y$
- Solution time and memory complexity is $O(|s|^2 + Q)$
  - Q — the number of queries

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Suffix tree or automaton solution

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- Still $O(|s|^2)$ time and $O(|s|^2)$ memory solution

C. Crime

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- Find number of different permutations
  - Of length n
  - Every two consecutive elements are coprime

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- Based on these equivalence classes count number of different multisets of classes
  - There are 1 728 000 of those

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    - There are 1 728 000 of those
- Make dynamic programming

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- Color each ball x, such that $l \leqslant x \leqslant r$, to black

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- You are given n white balls
- In one turn you choose (l, r), so that $1 \leqslant l \leqslant r \leqslant n$ equiprobably
- Color each ball x, such that $l \leqslant x \leqslant r$, to black
- What is the expected number of turns, so that every ball is colored?

D. Endless spin

Solution

- Answer is $\sum_{i=0}^{+\infty} p(i)$
  - $p(x)$ is the probability, that in x moves there exists a white ball

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- Answer is $\sum_{i=0}^{+\infty} p(i)$
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- How to calculate $p(x)$?

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  - d_A is the probability to leave A white in x moves

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    - Choosing these intervals will color some subset of $\overline{A}$

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  - Use inclusion-exclusion formula

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  - c_A is the number of intervals that cover only balls from $\overline{A}$
  - Then $d_A = \left(\frac{c_A}{\binom{n+1}{2}}\right)^x$
- So answer is $\sum_{i=0}^{+\infty} \left(1 - \sum_A (-1)^{|A|} \left(\frac{c_A}{\binom{n+1}{2}}\right)^i\right)$

D. Endless spin

Solution

- We know that $d_\varnothing = 1$, so:
  - Answer is: $\sum_{i=0}^{+\infty} \sum_{A \neq \varnothing} (-1)^{|A|} \left(\frac{c_A}{\binom{n+1}{2}} \right)^i$
- Change the order $\sum_{A \neq \varnothing} (-1)^{|A|} \sum_{i=0}^{+\infty} \left(\frac{c_A}{\binom{n+1}{2}} \right)^i$

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  - It's 2^n geometric series sums

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- Change the order $\sum_{A \neq \varnothing} (-1)^{|A|} \sum_{i=0}^{+\infty} \left(\frac{c_A}{\binom{n+1}{2}} \right)^i$
  - It's 2^n geometric series sums
- For every c_A and $(|A| \bmod 2)$ calculate the number of such A

D. Endless spin

Solution

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  - It's 2^n geometric series sums
- For every c_A and ($|A|$ mod 2) calculate the number of such A
- As an exercise come up with dynamic programming polynomial solution to do that

E. JZPTREE

- You are given a tree

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  - Number of vertices is at most 50 000

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  - Number of vertices is at most 50 000
- Calculate for each vertex v:
  - $\sum_u d_{v,u}^k$
  - $d_{x,y}$ is the distance from v to u
  - k is up to 50

E. JZPTREE

Solution

- Formula using Stirling numbers of second kind:
  - $d^k = \sum_{i=0}^{k} S(k,i) \cdot d \cdot (d-1) \cdot \dots \cdot (d-i+1)$
  - $d^k = \sum_{i=0}^{k} S(k,i) \cdot \binom{d}{i} \cdot i!$
  - $S(k,i)$ is the Stirling number of second kind
    - The number of ways to color k element set into i colors

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Solution

- Formula using Stirling numbers of second kind:
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- So for every vertex v calculate array a:
  - $a_i = \sum_{u} \binom{d_{v,u}}{i}$

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  - $S(k,i)$ is the Stirling number of second kind
    - The number of ways to color k element set into i colors
- So for every vertex v calculate array a:
  - $a_i = \sum_{u} \binom{d_{v,u}}{i}$
- To add one edge, one has to increase every $d_{v,u}$ by one
  - $\binom{d+1}{i} = \binom{d}{i} + \binom{d}{i-1}$
  - $a_i^{\mathtt{new}} = a_i + a_{i-1}$

E. JZPTREE

Solution

- To calculate *a* first make tree rooted
  - Sum up all $\binom{d}{i}$ over all descendants first
  - Then sum up all $\binom{d}{i}$ for not descendants by second DFS

E. JZPTREE

Solution

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- Calculate a for all vertices in $O(nk)$ time
- Calculate $S(i, j)$ and $i!$
- Use formula to get answer for every vertex

F. Jinkeloid

- You are given a string s of length up to 10^5

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  - Given $c_1, c_2, \dots c_k$ — letters
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- String consists of first 20 letters of alphabet
- Answer queries:
  - Given $c_1, c_2, \ldots c_k$ — letters
    - $k \leqslant 5$
  - Find number of pairs (i, j), so that $s(i, j)$ contains even number of each of these k letters

F. Jinkeloid

Solution

- For each prefix $0 \leqslant i \leqslant |s|$ find subset p_i:
  - which letters enter odd number of times

F. Jinkeloid

Solution

- For each prefix $0 \leqslant i \leqslant |s|$ find subset p_i:
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- For substring $s(i+1, j)$ we have to calculate $p_i \oplus p_j$

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- Calculate f_A — number of i such that $p_i = A$
- Calculate $g_A = \sum\limits_{A \subset B} f_B$
  - It's just partial sums on $2 \times 2 \times \ldots \times 2$ array
  - Calculated in $O(2^{|\Sigma|}|\Sigma|)$

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Solution

- To answer the queries:
  - p_i and p_j have to have equal parity for letters in query

F. Jinkeloid

Solution

- To answer the queries:
  - p_i and p_j have to have equal parity for letters in query
  - For each X of 2^k parities of given k letters get g_A
    - A contains only letters from query
    - Calculate $d_X = g_A$

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        - Calculate $d_X = g_A$
    - Use inclusion-exclusion formula
    - `for` $X = (2^k - 1) \ldots 0$:

      `for` $Y \supset X$:

      $d_X := d_X - d_Y$
    - d_X is number of p_i, so that given letters' parity is X and the other letters' parity is either odd or even

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    $d_X := d_X - d_Y$
  - d_X is number of p_i, so that given letters' parity is X and the other letters' parity is either odd or even
  - Answer is $\sum\limits_X \frac{d_X(d_X-1)}{2}$

G. Unsolvable Problem

- Given n up to 10^9
- Find positive a and b such that
  1. $a + b = n$
  2. $\mathrm{lcm}(a, b)$ is maximum possible

G. Unsolvable Problem

Solution

- If $x > y$ and $d > 0$, then $xy \geqslant (x + d)(y - d)$

G. Unsolvable Problem

Solution

- If $x > y$ and $d > 0$, then $xy \geqslant (x+d)(y-d)$
- p equal to smallest prime greater than $\frac{n}{2}$
  - It's coprime to $n-p$, since $n-p < p$

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  - So answer is not less than $(n - p)p$
  - You don't have to look to $x > p$
  - Gap between prime numbers is small enough to try every $\frac{n}{2} < x \leqslant p$

H. Pieces

- Given a string of length not greater than 16
- In one move you can erase any subsequence, that is palindrome
- Find minimum number of moves to erase all string

H. Pieces

Solution

- For every of $2^n - 1$ subsequences calculate if it's palindrome

H. Pieces

Solution

- For every of $2^n - 1$ subsequences calculate if it's palindrome
  - Let P be the set of all palindrome subsequences

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- $f[A] = \min_{B \in P \wedge B \subset A} f[B] + 1$

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Solution

- For every of $2^n - 1$ subsequences calculate if it's palindrome
  - Let P be the set of all palindrome subsequences
- $f[A]$ — minimum number of moves to erase subset A
- $f[A] = \min\limits_{B \in P \wedge B \subset A} f[B] + 1$
- Calculated in $O(3^n)$

I. Burning

- You are given triangles
- For every k find the area of a plane covered by exactly k triangles

I. Burning

Solution

- Intersect all pairs of sides of all triangles

I. Burning

Solution

- Intersect all pairs of sides of all triangles
- Get all x-coordinates of all intersection and all vertices
  - $x_1 < x_2 < \cdots < x_k$ be those coordinates

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- Intersection of this part of plane with every triangle is either empty set or trapezoid

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  - $x_1 < x_2 < \cdots < x_k$ be those coordinates
- Consider the part of plane with points (x, y) such that $x_i < x < x_{i+1}$
- Intersection of this part of plane with every triangle is either empty set or trapezoid
- No two non-vertical trapezoid sides intersect

I. Burning

Solution

- Intersect every side of triangle with this part of the plane

I. Burning

Solution

- Intersect every side of triangle with this part of the plane
- Get middle point of intersection

I. Burning

Solution

- Intersect every side of triangle with this part of the plane
- Get middle point of intersection
- Sort all segments by y-coordinate of this middle point

I. Burning

Solution

- Intersect every side of triangle with this part of the plane
- Get middle point of intersection
- Sort all segments by y-coordinate of this middle point
- Do another sweepline iterating over segments

I. Burning

Solution

- Intersect every side of triangle with this part of the plane
- Get middle point of intersection
- Sort all segments by y-coordinate of this middle point
- Do another sweepline iterating over segments
- Each segment is either the start of a triangle or the end
    - Keep track of k — number of triangles covering

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Solution

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- Sort all segments by y-coordinate of this middle point
- Do another sweepline iterating over segments
- Each segment is either the start of a triangle or the end
    - Keep track of k — number of triangles covering
- Calculate trapezoid area and add it to corresponding answer

J. No Pain No Game

- You are given n natural numbers ($n \leqslant 50\,000$)

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  - Each consists of L and R

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  - Each consists of L and R
  - Find pair (i, j) such that $i \neq j$ and $L \leqslant i, j \leqslant R$

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    - And $\gcd(a_i, a_j)$ is maximum possible

J. No Pain No Game

Solution

- Let's answer all queries in order of increasing R

J. No Pain No Game

Solution

- Let's answer all queries in order of increasing R
  - Consider gcd is equal to v

J. No Pain No Game

Solution

- Let's answer all queries in order of increasing R
    - Consider gcd is equal to v
    - Consider rightmost two positions $i < j \leqslant R$:
        - so that $v \mid a_i$ and $v \mid a_j$

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- Let's answer all queries in order of increasing R
  - Consider gcd is equal to v
  - Consider rightmost two positions $i < j \leqslant R$:
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  - For every $L \leqslant i$ answer is at least v
  - For every $1 \leqslant i \leqslant R$ store the maximum possible divisor of a_i

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  - Consider rightmost two positions $i < j \leqslant R$:
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  - For every $L \leqslant i$ answer is at least v
  - For every $1 \leqslant i \leqslant R$ store the maximum possible divisor of a_i
    - Such v, so that there is $j > i$ and $j \leqslant R$
    - And $v \mid a_j$

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  - Keep track of interval tree or binary indexed tree, say t

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  - Keep track of last number, that is divisible by each v

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    - And $v \mid a_j$
  - Keep track of interval tree or binary indexed tree, say t
  - Keep track of last number, that is divisible by each v
  - To increase R by one, iterate over all $v|a_R$
    - Make `t[last[v]] := max(t[last[v]], v)`
    - Update `last[v] := R`

J. No Pain No Game

Solution

- Let's answer all queries in order of increasing R
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  - Keep track of last number, that is divisible by each v
  - To increase R by one, iterate over all $v|a_R$
    - Make `t[last[v]] := max(t[last[v]], v)`
    - Update `last[v] := R`
  - Answer for query (L, R) is maximum in `t[L..R]`

K. Sad Love Story

- You are given sequence of n points
- n is at most $5 \cdot 10^5$

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- You are given sequence of n points
- n is at most $5 \cdot 10^5$
- Points are generated pseudorandomly
  - With coordinates up to n
- After each point answer, what is the distance between closest two points?

K. Sad Love Story

Solution

- Maintain sorted by x-coordinate array of all already added points

K. Sad Love Story

Solution

- Maintain sorted by x-coordinate array of all already added points
- When new (x_0, y_0) point added:
  - Let d be the current answer

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- Maintain sorted by x-coordinate array of all already added points
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  - Check all points (x, y) such that $|x - x_0| < d$
    - Other points are not closer than d

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Solution

- Maintain sorted by x-coordinate array of all already added points
- When new (x_0, y_0) point added:
  - Let d be the current answer
  - Check all points (x, y) such that $|x - x_0| < d$
    - Other points are not closer than d
  - Update answer by distance to these points

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Solution

- Intuitively the runtime is explained like this:
  - Consider we added p points to our set

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Solution

- Intuitively the runtime is explained like this:
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  - Let's make $r \times r$ grid of $[1 \dots n] \times [1 \dots n]$ square

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Solution

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  - Consider we added p points to our set
  - Let's make $r \times r$ grid of $[1 \dots n] \times [1 \dots n]$ square
    - Choose r such that the probability of two points locating in the same cell is at least $\frac{1}{2}$

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  - Let's make $r \times r$ grid of $[1 \ldots n] \times [1 \ldots n]$ square
    - Choose r such that the probability of two points locating in the same cell is at least $\frac{1}{2}$
    - Birthday paradox says that number of cells can be quadratic of p, so $r \approx p$

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  - So expected number of points in $x_0 - d < x < x_0 + d$ is $\frac{2pd}{n}$

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  - So expected number of points in $x_0 - d < x < x_0 + d$ is $\frac{2pd}{n}$
  - $d \approx \frac{n}{r} \approx \frac{n}{p}$
    - Expected number of points is $\frac{2pd}{n} \approx 2\frac{\frac{n}{p}p}{n} = 2$

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  - So expected number of points in $x_0 - d < x < x_0 + d$ is $\frac{2pd}{n}$
  - $d \approx \frac{n}{r} \approx \frac{n}{p}$
    - Expected number of points is $\frac{2pd}{n} \approx 2\frac{\frac{n}{p}p}{n} = 2$
  - So summing up over all p, we get $O(n)$ runtime