Problem A

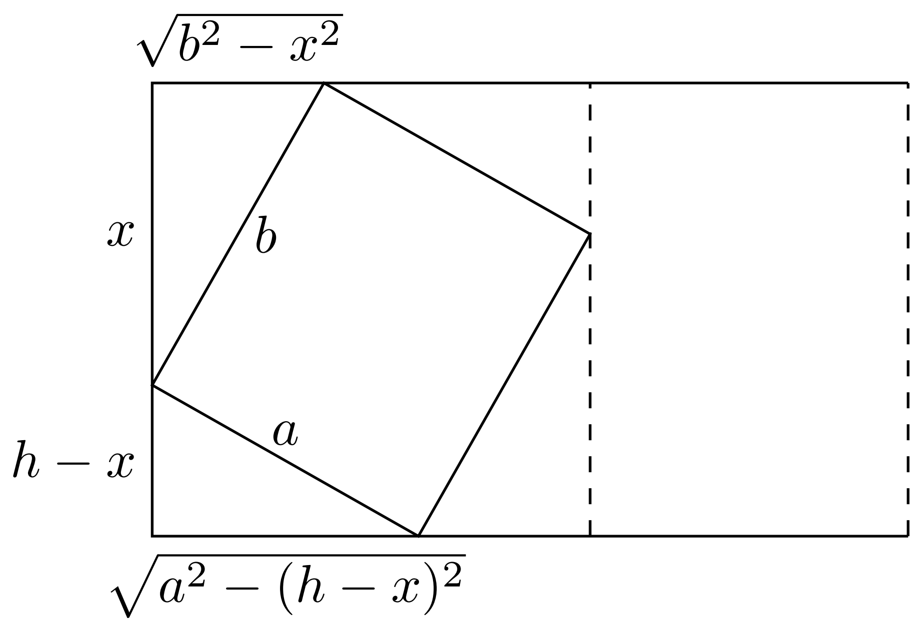
When area of a triangle is integer?

$$\overrightarrow{(b-a)} \times \overrightarrow{(c-a)} \mod 2 = 0$$

Lets replace coordinates of all points by their values modulo 2. Then answer will be the same and we will have only 4 different point, each of them can be multiple times. Now we just try to select three different points from our weighted points. If area is integer then we add to the answer multiplication of corresponding binomial coefficients.

Problem B

First, lets try to put postcard so that its side would be parallel to the sides of envelope. If we can't do that then picture looks like this:



Lets write similarity property of our triangles:

$$\frac{\sqrt{b^2 - x^2}}{b} = \frac{h-x}{a}$$

If we square each part then we will get simple quadratic equation on x.
In the end we can put postcard inside envelope if $w \geq \sqrt{b^2 - x^2} + \sqrt{a^2 - (h-x)^2}$.

Problem C

We can rotate and intersect our rectangles manually, but there is simple formula if we write all similarity and equality property for our triangles.

Problem D

If segment doesn't intersect with the circle, then answer is the length of the segment. Otherwise answer is sum of lengths of tangent segments $\sqrt{d_1^2 - r^2} + \sqrt{d_2^2 - r^2}$ and the length of an arc:

$$r \cdot (\mathrm{Angle}(H_1 - C, H_2 - C) - (\pi/2 - \mathrm{asin}(r/d_1)) - (\pi/2 - \mathrm{asin}(r/d_2))).$$

And you actually don't need to find points in this problem.

Problem E

Lets imagine that we are doing binary search by answer, checking that it is $\leq D$. Then we need to find equilateral triangle with vertices inside three circles with centers in our points and radius D.

Lets look at the set of all points that are corresponds to the third vertex of the triangle if first vertex is fixed and second one lies inside a circle. Obviously answer is this circle (disc) rotated 60 degrees around first point.

Lets look at the set of all points that are corresponds to the third vertex of the triangle if first vertex lies inside first circle and second one lies inside of a second circle. Answer is the circle with radius $2D$ and with center in the second point, rotated 60 degrees around first. All that left — intersect this circle with out third circle.

But after all this statement we can get rid of the binary search and say that answer is

$$D = \mathrm{dst}(P_3, P_1 + \mathrm{rot}(P_2 - P_1, 60))/3.$$

Problem F

Lets look at the line that is the answer. Lets shift it until one of the points will lie on our line. Lets fix that point. After that we will start to rotate our line and lets look how answer will change.

Each segment will add +1 to a certain segment of angles. So we have events for opening and for closing segments. We sort them and calculate answer. We'll get in the end $\mathcal{O}(n^2 \log n)$.

Problem G

Lets denote distances as x, y and z. Lets select the person who we want to make a winner. He wins each other so we will get system of such comparisons:

$$\frac{x}{vx_i} + \frac{y}{vy_i} + \frac{z}{vz_i} < \frac{x}{vx_0} + \frac{y}{vy_0} + \frac{z}{vz_0}$$

Note that if we will divide each distance by some d, then winner will be the same. Lets divide them by z:

$$x' \left(\frac{1}{vx_i} - \frac{1}{vx_0} \right) + y' \left(\frac{1}{vy_i} - \frac{1}{vy_0} \right) + \left(\frac{1}{vz_i} - \frac{1}{vz_0} \right) < 0$$

And this is just half-planes intersection problem. We can do it in $\mathcal{O}(n^3)$. And in the end we will get $\mathcal{O}(n^4)$.