Problem K. Irreducible Polynomials

Input file:	`standard input`
Output file:	`standard output`
Time limit:	1 second
Memory limit:	256 mebibytes

There is prime number p. $Z_p = \{0, 1, \ldots, p-1\}$ set of integers modulo p. At this field multiplication and addition operations are done modulo p. Now consider irreducible monic polynomials over this field of a kind:

$$f(x) = x^n + a_{n-1}x^{n-1} + \ldots a_1 x + a_0,$$

where n is power of the polynominal, x is variable, $a_i \in Z_p$ are coefficients. Value x which makes $f(x)$ equal to zero is a root of this polynomial. Let's expand the field, so any polynomial of any power, which is no less than one, has at least one root.

Monic polynomial is irreducible, if it has a root which is not a root of any monic polynomial of lower power with coefficients from Z_p.

For example, polynomial $x^2 + x + 1$ in the field Z_2 irreducible. Its root (labeled as e_2) is not a root of polynomial x, nor $x + 1$. Same can be said about second root $1 + e_2$ of specified polynomial. And this is only irreducible polynomial in the set Z_2.

Your task is to determine amount of irreducible polynomials of power n in the set Z_p.

Limits

Numbers p, n, m are integers, p is prime number.

$1 \le p, n \le 10^9$, $1 \le p^n < 10^{18}$.

Input

The only line of input file contains three numbers p, n.

Output

Output amount of irreducible polynomials of power n in the field Z_p.

Example

standard input	standard output
2 2	1
3 4	18

Problem L. Game

Input file:	`standard input`
Output file:	`standard output`
Time limit:	1 second
Memory limit:	256 mebibytes

Two players are playing following game. Each of them chooses a consequence, consistng of 0 and 1. Then they start tossing symmetric coin until results of the last tosses fit consequnce of one of the player (0 coreesponds to tail, 1 to coat). Of course, the player whose consequence is tossed earlier wins. You have to determine probabilty of the first player's victory by given consequences.

Limits

Consequences, chosen by players are not empty, are different and have length not exceeding 10.

Input

The first line contains consequence of the first player, the next one contains consequence of the second player (without spaces).

Output

Output probablity of the first player's consequence being tossed earlier, than the second player's consequence with accuracy not less than 10^{-8}.

Example

standard input	standard output
001 110	0.50000000
00 10	0.25000000

Problem M. Paint the Fence

Input file:	`standard input`
Output file:	`standard output`
Time limit:	2 seconds
Memory limit:	256 mebibytes

There is given a fence, consisting of N planks, following each other Problem is to paint it such way, that each plank is fully painted with one of C colors. Some pairs of colors are considered incompatible and this is unacceptable for two neighbouring planks to be painted with corresponding colors. There are M such pairs. Your task is to determine amount of acceptable paintings, i.e. such ones, which do not contain any two neighbouring planks painted with incompatible colours.

Limits

N, C, M are integers.

$1 \le N \le 10000$, $1 \le C \le 100$, $0 \le M \le C(C-1)/2$.

Input

The first line contains three integers N, C, M. Each of the following M lines contains two numbers, determening a pair of incompatible colors.

Output

Output remainder of division of acceptable fence paintings by $10^9 + 7$.

Example

standard input	standard output
2 3 2 1 2 2 3	5
3 2 1 1 2	2

Problem N. Cities

Input file:	`standard input`
Output file:	`standard output`
Time limit:	1 second
Memory limit:	256 mebibytes

N cities are located on coordinate line. It was decided to choose different K pairs of these cities and call cities of each pair sister cities. So, any city cannot have more than one sister-city.

You need to determine maximal and mininal possible total sum of distances between pairs of sister cities.

Limits

N, K are integers.

$1 \le N \le 10000$, $0 \le K \le N/2$. Absolute value of cities' coordinates doesn't exceed 10^9.

Input

The first line contains integers N and K. The second has N numbers, determing cities' coordinates.

Output

Output two numbers in the only line. These are maximal and minmal summary distance between sister cities in K pairs.

Example

standard input	standard output
5 2 0 3 4 7 9	13 3
3 1 2 7 5	5 2

Problem O. Vacation on a River

Input file:	`standard input`
Output file:	`standard output`
Time limit:	1 second
Memory limit:	256 mebibytes

A couple decided to spend their vacation on a bank of a river. George likes high places and wants to go to the spot, where the bank is as high as possible above river level. But his wife Mary is afraid of heights and wants to spend the vacation where it's as low, as possible. Now they are going in a car along a one-way main road, and there are n turns to the river along it. The road over each turn leads to the river, and both spouses know the height of the spot, where corresponding road leads. Of course, George drives the car, but Mary can distract George, so he doesn't notice one of the turns *except of the last one*. All the turns look so like each other, that George cannot be sure to which spot it leads. Main road continues over the last turn and leads to the spot with known bank height too. Obviously, George can use one of the following strategies: either he turns on first turn he noticed, on second, third etc., either not to turn at all (of course, in case if he notices less turns than required for his strategy, a couple will arrive at the place in the end of the main road).

Determine optimal strategy for George, supposing that Mary will act optimally too.

Limits

N, h_i are integers.

$1 \leq N \leq 5000$, $0 \leq h_i \leq 1000$.

Input

The first line contains number N. The second line has $N + 1$ numbers h_i, which determine the height of the spot, where the road from ith turn leads (h_{n+1} is height of the place, where the main road leads without any turns).

Output

Output maximum height of place, where George can get with optimal Mary's opposition in the first line. In the second line write $n+1$ number, which are probablities for George to use each of his clean strategies to reach the height. All the values have to be output with accuracy no less than 10^{-6}. In case if there are multiple optimal strategies, choose the one where probability of choice "Do not turn anywhere" is maximum. If there are multiple such strategies too, then choose the one with maximum probability of nth turn choice, etc.

Example

standard input	standard output
2 0 6 3	4.000000 0.333333 0.666667 0.000000
3 2 3 4 2	2.800000 0.400000 0.400000 0.200000 0.000000

Problem P. Representable numbers

Input file:	`standard input`
Output file:	`standard output`
Time limit:	2 seconds
Memory limit:	256 mebibytes

There are given two positive integers a and b. Number x is representable, if it can be represented as sum $x = x_1 + x_2 + \dots x_n$ of finite (probably null) number of summands x_i, each equal to a or b. Define, how many different numbers on sector $[A, B]$ are representable.

Limits

a, b, A, B are integers.

$0 \le a, b \le 10000$, $0 \le A \le B \le 10^7$.

Input

The only line contains numbers a, b, A, B.

Output

Output amount of representable through a and b numbers on segment $[A, B]$.

Example

standard input	standard output
4 5 7 12	4
6 10 20 30	6

Problem Q. Quadratic permutation

Input file:	`standard input`
Output file:	`standard output`
Time limit:	1 second
Memory limit:	256 mebibytes

Consider permutation of numbers from a to b. It is quadratic, if a perfect square of each element is its sum with one, that takes its place after permutation. More precisely, quadratic permutation is such bijection of p set of integers from a to be b on itself, that for any i $i + p(i) = j^2$ for some integer j. Find such quadratic permutation for given a and b.

Limits

a, b are integers.

$0 \le a \le b \le 20$.

Input

The only line contains integers a and b.

Output

Output $b - a + 1$ numbers, which determiune values $p(i)$ for all i from a to b, where p is some quadratic permutation. If there is no such permutation with given a and b, output one number -1.

Example

standard input	standard output
`1 9`	`8 2 6 5 4 3 9 1 7`
`3 5`	`-1`

Problem R. De Bruijn's Cycle

Input file:	`standard input`
Output file:	`standard output`
Time limit:	1 second
Memory limit:	256 mebibytes

De Bruijn's cycle of order n for set $D = \{0, 1, \ldots, b-1\}$ is such cyclyc sequence a_0, a_1, …, a_{l-1}, where each vector of length n over set D occurs in this sequence exactly once (i.e. for any b_0, b_1, …, $b_{n-1} \in D$ there is only one such k in limits from 0 to $l-1$, that $b_j = a_{(k+j) \bmod l}$ for all $j = \overline{0, n-1}$) Build such consequence.

Limits

n, b are integers.

$1 \le n \le 1000$, $1 \le b \le 10$, $b^n \le 10^4$.

Input

The only line contains numbers n and b.

Output

In the only line output De Bruijn's cycle of order n for set of b-based numbers (without spaces).

Example

standard input	standard output
`2 3`	`001102122`
`3 2`	`00010111`

Problem S. Card Duel

Input file:	`standard input`
Output file:	`standard output`
Time limit:	1 second
Memory limit:	256 mebibytes

Two players play a game. Each of players has initial amount of cards (n_1 and n_2 correspondently). On every turn players choose one card each and open it. Weaker card goes to retreat, the stronger one is taken back by the player who opened it. If the players have shown same cards, both go to retreat. The game is continued, until at least one of the players is over of cards. If one of the players still has at least a card when that happens, than he gets 1 point, and his rival 0. If both of the players are over of cards, than each gets 0.5 points. There are N kinds of cards totally. Strength relation of cards is nontransitive and defined with matrix A. If card i is stronger than j, A_{ij} is 1, and 0 otherwise. Define price of that game for the first player, supposing that second player plays optimally.

Limits

$1 \le n_1, n_2, N \le 8$.

$A_{ij} + A_{ji} = 1$ with $i \ne j$, $A_{ii} = 0$.

Input

The first line contains number N. Following N lines contain N numbers each, which define matrix A. Next line contains number n_1 and n_1 more numbers, each descripting a kind of corresponding card of the first player. The last line contains similar description of the second player's cards.

Output

Output price of the game for the first player with accuracy no less than 10^{-8}.

Example

<table>
<tr><th>standard input</th><th>standard output</th></tr>
<tr><td><pre>3
0 1 1
0 0 1
0 0 0
2 3 2
1 1</pre></td><td><pre>0.00000000</pre></td></tr>
<tr><td><pre>3
0 1 0
0 0 1
1 0 0
3 1 2 3
3 1 2 3</pre></td><td><pre>0.66666667</pre></td></tr>
<tr><td><pre>3
0 1 0
0 0 1
1 0 0
3 1 2 3
3 2 2 3</pre></td><td><pre>0.66666667</pre></td></tr>
</table>

Problem T. Plane Boarding

Input file:	`standard input`
Output file:	`standard output`
Time limit:	1 second
Memory limit:	256 mebibytes

N men are boarding to the plane, which has exactly N seats. Each of the passengers has a ticket for a seat in the plane, and there are no two similar tickets. But some of the passengers are mad. They are boarding the plane by one, and don't look at the ticket, but take a seat chosen of free ones irregulary. Normal people take seat, specified in the ticket. But if his seat is already taken, then he takes one of free seats with equal probability not to start a scandal. Define probability of taking a seat specified in the ticket for each passenger.

Limits

N is integer, $1 \leq N \leq 15$.

Input

The first line contains number N. The second one contains N numbers, each defining corresponding passenger in the order of boarding the plane (0 defines normal man, 1 defines mad one)

Output

Output N numbers, each descripting probability that corresponding passengeer will take his place. All the values have to be output with accuracy no less than 10^{-8}.

Example

standard input	standard output
3 0 1 0	1.00000000 0.5000000 0.50000000
4 1 0 1 0	0.25000000 0.75000000 0.33333333 0.33333333