GCPC 2014

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acm International Collegiate Programming Contest

The Problem Set

Good luck and have fun!

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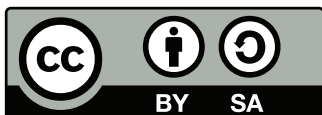
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Problem A
Algebraic Teamwork

The great pioneers of group theory and linear algebra want to cooperate and join their theories. In group theory, permutations – also known as bijective functions – play an important role. For a finite set A, a function $\sigma : A \to A$ is called a permutation of A if and only if there is some function $\rho : A \to A$ with

$$\sigma(\rho(a)) = a \text{ and } \rho(\sigma(a)) = a \quad \text{for all } a \in A.$$

The other half of the new team – the experts on linear algebra – deal a lot with idempotent functions. They appear as projections when computing shadows in 3D games or as closure operators like the transitive closure, just to name a few examples. A function $p : A \to A$ is called idempotent if and only if

$$p(p(a)) = p(a) \quad \text{for all } a \in A.$$

To continue with their joined research, they need your help. The team is interested in non-idempotent permutations of a given finite set A. As a first step, they discovered that the result only depends on the set's size. For a concrete size $1 \le n \le 10^5$, they want you to compute the number of permutations on a set of cardinality n that are **not** idempotent.

Input

The input starts with the number $t \le 100$ of test cases. Then t lines follow, each containing the set's size $1 \le n \le 10^5$.

Output

Output one line for every test case containing the number modulo $1\,000\,000\,007 = (10^9 + 7)$ of **non**-idempotent permutations on a set of cardinality n.

Sample Input	Sample Output
3	0
1	1
2	6425
2171	

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Problem B

Beam me out!

King Remark, first of his name, is a benign ruler and every wrongdoer gets a second chance after repenting his crimes in the Great Maze!

Today's delinquent is a renowned computer scientist, but his fame didn't do him any good after he declined to do research on the so called and soon-to-be-famous Remark's algorithms! Those strange randomized algorithms may run indefinitely long (or even never terminate) and may or may not produce a right answer if terminated.

Handily, the Great Maze got recently a major upgrade with the newest beaming technology which made all doors obsolete: After the delinquent says the magic words "I was wrong and will never disappoint king Remark again!" he will be immediately beamed to the next room. It will be chosen randomly from a list of possible goal rooms.

The Great Maze consists of n rooms numbered 1 to n. Every detainee starts his quest for pardon in room 1 and hopes to get to the throne room n in which he will receive his pardon. If he ends up in a room, whose list of goal rooms is empty, his tour is over; through he could surely say the magic words again and again – that would not hurt, but would not help him either.

Great king Remark, as most of the kings, doesn't like surprises and summoned you to answer two questions: Is it guaranteed, that the criminal will get to the throne room and is there a limit of beaming operations after which the game is over for sure.

You know better, than to disappoint the great king with a wrong answer or no answer at all, don't you?

Input

The input contains a single test case. It starts with a line consisting of an integer $2 \leq n \leq 50\,000$ – the number of rooms in the Great Maze. For each of the rooms 1 to $n - 1$, two lines will follow representing the corresponding list of the goal rooms (in order 1 to $n - 1$). Bear in mind, that after reaching the throne room n the quest is over. Thus, the list of the throne room is not a part of the input.

The first of these two lines will contain an integer $0 \leq m \leq n$ – the number of goal rooms on the list. The second line will contain a list of m goal rooms or an empty string, if $m = 0$. Each list will be sorted in strictly ascending order (this implies every number on the list will be unique) and consist from integers between 1 and n, inclusive.

The total number of goal rooms summed over all lists will not exceed 10^6.

Output

For each test case a line consisting of two words:

- the first word must be "PARDON", if the probability for the prisoner getting to the throne room during his random walk is 100%, or "PRISON" otherwise.

- the second word must be "LIMITED", if a limit for the number of beaming operations exists, or "UNLIMITED" otherwise.

Sample Input I	Sample Output I
3	PARDON LIMITED
2	
2 3	
1	
3	

Sample Input II	Sample Output II
3	PRISON LIMITED
2	
2 3	
0	

(More sample inputs follow on the next page.)

Sample Input III

```
3
2
2 3
2
1 3
```

Sample Output III

```
PARDON UNLIMITED
```

Sample Input IV

```
3
2
2 3
1
2
```

Sample Output IV

```
PRISON UNLIMITED
```

Problem C

Bounty Hunter

Spike is a bounty hunter and he is currently tracking a criminal! To investigate he uses his spaceship, the Swordfish II, and travels to N different places on 2D Euclidean space before returning to his crew at the starting location with all the information he has gathered. The starting location is the leftmost place (with the lowest x-coordinate) and Spike wants to travel to *every* other place before returning. However space fuel costs a lot of Woolongs and Spike would rather spend his money on special beef with bell peppers. Therefore he wants to travel the minimum possible distance.

On top of that he is being chased by the Red Dragon crime syndicate. To make sure they don't catch him he can only visit places in increasing order of their x-coordinate until he reaches the rightmost place (with the largest x-coordinate), then he can turn around and visit places in decreasing order of their x-coordinate until he reaches his starting location again.

Input

The input starts with an integer T ($1 \leq T \leq 100$) specifying the number of test cases that follow. Each test case consists of an integer N ($2 \leq N \leq 512$) specifying the number of places in the tour. The coordinates of these places are given as integers in the next N lines, x-coordinate first, y-coordinate second ($0 \leq x, y \leq 5000$). The places are given in ascending order of the x-coordinate. Every place has a unique x-coordinate.

Output

For each test case, output on a single line the minimum travel distance needed to complete the tour. Your output should have an absolute or relative error of at most 10^{-2}.

Sample Input	Sample Output
2	9.300563079746
5	400
0 1	
1 2	
2 0	
3 2	
4 1	
3	
100 1	
200 1	
300 1	

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Problem D

Connected Caves

You're an intern at *Greedy Cave Plundering Corporation*. Your new job is to help with one of *GCPC*'s projects: extracting valuable gemstones from a network of connected caves.

The history of the project so far:

- Tom, one of the project managers, has hired Jill, a gemstone expert. She has already surveyed these caves and determined the value of all gemstones in each cave.

- Ruby, another project manager (PM), has been busy "guesstimating" a time frame, the budget and possible profits for this project.

- Evelyn and Jerry (also PMs) have hired two cave diggers and instructed them to extract the gemstones.

- The cave diggers needed equipment to extract the gemstones and transport them to the surface. So Jimmy (another PM) ordered some machines. Sadly, he did not communicate well with the cave diggers - he bought machines that are way too big for the current passage ways between caves.

- To fix this, Jimmy was fired and James, his successor, has ordered a very big drilling machine, so the cave diggers can widen the passage ways. However, James was fearful of cutting into his salary bonus, so he bought the cheapest machine he could find. Soon, after widening the passage way from the surface into cave 1, the cave diggers found out that the new drilling machine is very heavy and not so powerful, so it can only be used to widen a passage way that goes downwards. Once it has reached a lower cave, it can not be carried back up because of its weight.

- This is when Alice, the project manager in charge of budget planning, announced she would ignore further requests for more equipment.

- In an eight hour meeting discussing the problems of the project, it was decided to limit the project expenses. So, if further drilling would hurt profits or if the drilling machine reaches a cave where it can not go on, it will just be abandoned. Using the other machines, the cave diggers will then be able to harvest at least some of the gemstones. Afterwards, the dig site will probably be sold to another company. Ruby is already thinking about hiring her brother as a sales expert...

This is where you come in. Tom has hired you to determine which passage ways should be widened and thereby which caves should be visited to maximize profits. Widening a passage way requires energy, materials and so on. You have to consider these costs. You will be given a map of all caves and passage ways. The map was created by the cave diggers, so it should be accurate. It contains only passage ways the drilling machine can handle, including the correct drilling direction. All caves in the map are reachable from cave one.

Input

The input starts with a line containing T, the number of test cases $(1 \leq T \leq 10)$.
Each test case starts with a line containing two integers: N, the number of caves, and E, the number of passage ways $(1 \leq N \leq 2 \cdot 10^4; 0 \leq E \leq 10^5)$.
The second line contains N integers $v_1 \ v_2 \ldots v_N$. v_i describes the value of all gemstones in cave i $(0 \leq v_i \leq 10^4)$.
Each of the next E lines contains three integers: $a_e \ b_e \ c_e$. Such a line represents one of the possible direct passage ways from cave a_e to cave b_e, widening this passage way will cost c_e $(1 \leq a_e, b_e \leq N;$ $0 \leq c_e \leq 10^4)$. It is guaranteed that the input contains only valid digging directions, i.e. cave b_e will be strictly deeper than a_e.
You always start from cave number 1, because the digging machine is already there and there are no caves above.

Output

Output two lines for every test case. In the first line print two integers P and C, where P is the profit of the best possible route from top to bottom and C is the number of visited caves on that path. In the second line print the IDs of those caves, ordered from top to bottom.
If there are multiple solutions with optimal profit, print any.

Sample Input	Sample Output
3	10 1
1 0	1
10	17 2
4 3	1 3
10 20 30 40	50 3
1 2 19	1 3 4
1 3 23	
1 4 34	
4 4	
10 20 30 40	
1 2 10	
2 4 20	
1 3 20	
3 4 10	

Problem E

Equator

In a galaxy far away, the planet *Equator* is under attack! The evil gang *Galatic Criminal People Cooperation* is planning robberies in Equator's cities. Your help is needed! In order to complete your training for becoming a lord of the dark side you should help them deciding which cities to rob.

As the name says, the desert planet Equator only can be inhabited on its equator. So the gang lands there at some point and travels into some direction robbing all cities on their way until leaving the planet again.
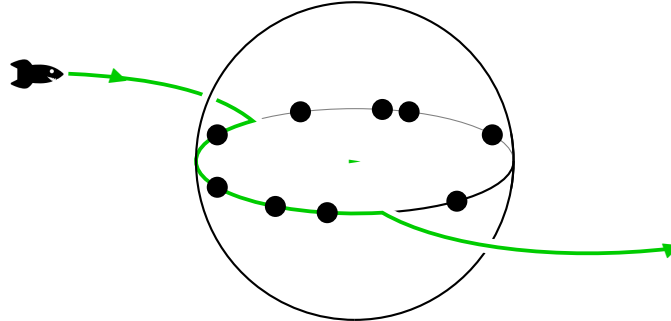
But what is still open for them is to decide where to land, which direction to take, and when to leave. Maybe they shouldn't even enter the planet at all? They do not consider costs for traveling or for running their ship, those are peanuts compared to the money made by robbery!

The cities differ in value: some are richer, some are poorer, some have better safety functions. So the gang assigned expected profits or losses to the cities. Help them deciding where to begin and where to end their robbery to maximize the money in total when robbing **every** city in between.

Input

The input starts with the number of test cases $T \le 30$. Each test case starts a new line containing the number of cities $1 \le n \le 1\,000\,000$. In the same line n integers c_i follow. Each c_i ($0 \le i < n$, $-1000 \le c_i \le +1000$) describes the money obtained when robbing city i, a negative c_i describes the amount of money they would lose.

Output

For each test case print one integer describing the maximum money they can make in total.

Sample Input	Sample Output
3	6
3 1 2 3	14
8 4 5 -1 -1 1 -1 -1 5	0
2 -1 -1	

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Problem F

Gold Rush

Alice and Bob are on an adventure trip. Deep in the woods they discover a mysterious deep cave which they enter flutteringly. They find an old console with a giant bar of gold in it. On the bar, there is a number n. Both tried to carry the gold out the cave, but it was still to heavy for one of them.

Suddenly a little fairy appears in the corner of the cave and approaches Alice and Bob: "This gold is heavy. It weights 2^n femto-grams (10^{-15}) and n can reach 62."

Bob answered: "What luck! Alice's knapsack can carry up to a femto-grams and mine b femto-grams with $a + b = 2^n$." Alice interjected: "But how can we divide the gold?"

Fairy: "I can help you with a spell that can burst one piece of gold into two equally weighted ones. But for each single spell, the cave will be locked one additional day."

Alice consults with Bob to use the help of the fairy and take all of the gold. How long will they be trapped if they are clever?

Input

The input starts with the number $t \leq 1000$ of test cases. Then t lines follow, each describing a single test case consisting of three numbers n, a and b with $a, b \geq 1$, $a + b = 2^n$, and $1 \leq n \leq 62$.

Output

Output one line for every test case with the minimal number of days that Alice and Bob are locked in the cave.

| Sample Input	Sample Output
3	1
2 2 2	2
2 1 3	7
10 1000 24	

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Problem G

Jewelry Exhibition

To guard the art jewelry exhibition at night, the security agency has decided to use a new laser beam system, consisting of sender-receiver pairs. Each pair generates a strip of light of one unit width and guards all objects located inside the strip. Your task is to help the agency and to compute for each exhibition room the minimum number of sender-receiver pairs which are sufficient to protect all exhibits inside the room.

Any room has a rectangle shape, so we describe it as an $[0, N] \times [0, M]$ rectangle in the plane. The objects we need to guard are represented as points inside that rectangle. Each sender is mounted on a wall and the corresponding receiver on the opposite wall in such a way that the generated strip is a rectangle of unit width and length either N or M. Since the new laser beam system is still not perfect, each sender-receiver pair can only be mounted to generate strips the corners of which have integer coordinates. An additional drawback is that the sender-receiver pairs can protect only items inside the strips, but not those lying on their borders. Thus, the security agency arranged the exhibits in such a way that both coordinates of any point representing an exhibit are non-integers. The figure below (left) illustrates eight items arranged in $[0, 4] \times [0, 4]$ (the second sample input). In the room, up to eight sender-receiver pairs can be mounted. The figure to the right shows an area protected by three sender-receiver pairs.
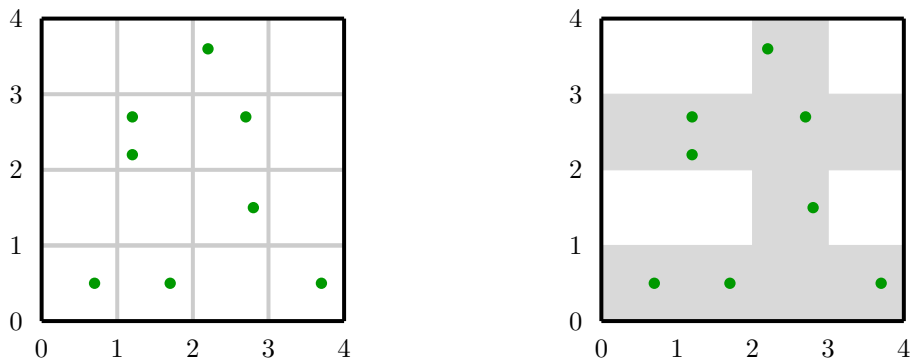
Input

The input starts with the number of exhibition rooms $R \leq 10$. Then the descriptions of the R rooms follow. A single description starts with a single line, containing three integers: $0 < N \leq 100$, $0 < M \leq 100$, specifying the size of the current room and $0 < K \leq 10^4$, for the number of exhibits. Next K lines follow, each of which consists of two real numbers x, y describing the exhibit coordinates. You can assume that $0 < x < N$, $0 < y < M$ and that x and y are non-integer.

Output

For every room output one line containing one integer, that is the minimum number of sender-receiver pairs sufficient to protect all exhibits inside the room.

Sample Input	Sample Output
2	1
1 5 3	3
0.2 1.5	
0.3 4.8	
0.4 3.5	
4 4 8	
0.7 0.5	
1.7 0.5	
2.8 1.5	
3.7 0.5	
2.2 3.6	
2.7 2.7	
1.2 2.2	
1.2 2.7	

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Problem H
JuQueen

JuQueen is the super computer with the best performance allover Germany. It is on rank 8 in the famous top500 list with its 458 752 cores. It draws a lot of energy (up to 2 301 kW), so we want to reduce that by underclocking the unused cores.

The cluster scheduling algorithm which is in charge of distributing jobs over the nodes and cores of a cluster will issue the following speedstepping commands:

- `change X S` changes the frequency of core `X` by `S` steps

- `groupchange A B S` changes the frequency of every core in range `[A,B]` by `S` steps

- `state X` returns the current state of core `X`

To be safe for the future, your program should be able to handle 4 587 520 cores. The initial frequency for each core is 0.

Input

The input contains a single test case. It starts with a line containing three integers C, N, and O, where C is the number of cores ($1 \leq C \leq 4\,587\,520$) to manage, N is the number of frequency steps for each core ($1 \leq N \leq 10\,000$) and O is the number of operations in the test program ($1 \leq O \leq 50\,000$). Then follow O lines, each containing one command as described above. X, A and B are 0-based IDs of the cores ($0 \leq A, B, X < C$; $A \leq B$). S is an integer number of steps, possibly negative ($-N \leq S \leq +N$).

Both, the `change` and the `groupchange` command will increase (or decrease) in single steps and stop as soon as **one** core in the group reaches the minimal (0) or maximal frequency (N).

Output

Output one line for every operation in the input. For `change` and `groupchange` print the changed number of steps, for `state` print the current state.

Sample Input I	Sample Output I
10 10 5	0
state 0	7
groupchange 2 9 7	7
state 9	3
groupchange 0 2 10	-3
change 0 -5	

Sample Input II	Sample Output II
4587520 10000 5	9950
groupchange 0 4587010 9950	42
groupchange 23 4587000 42	-1000
groupchange 4710 4587001 -1000	8992
state 1234560	1008
groupchange 6666 3060660 10000	

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Problem I

Laser Cutting

Jakob's laptop broke down, so he decided to build a new one by himself. After getting all the electronic parts, he needs to build a case. He decides to build one out of plywood, cut into pieces and glued together. He learns that, using the laser cutter of the FAU FABLAB, he can cut plywood sheets simply by creating a vector drawing. After some work, he has made a vector drawing. However, his drawing and calculating skills are rather poor, so he wants you to check his drawing.
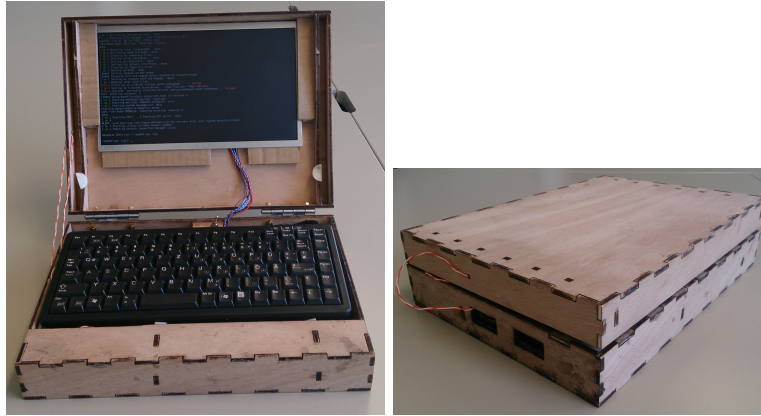

Figure 1 − Jakob's Laptop

The drawing consists of a number of polylines that are supposed to be non-self-intersecting polygons (which you have to check). To make things easier, all the lines that make up the polylines are parallel to either the x- or the y axis. Also, two polygons should not touch or intersect. Lastly, one part may contain some holes for plugging in other parts or electronics. However, a hole does not contain other parts or other holes. As parts and holes are both described by polygons, this means that any one polygon may be inside one other polygon, but not inside two other polygons.

Input

The input starts with the number of test cases t (with $t \leq 10$), on a line. Each test case starts with the number of polylines p (with $p < 50$), on a line. Every polyline starts with the number of coordinates n, with $5 \leq n \leq 50$. The next line contains $2n$ numbers, the x and y coordinates of the points on the polyline. You may assume that, given any two neighbouring points of a polyline, exactly one of their coordinates differ, i.e. all parts of a polyline are parallel to either x or y axis, and have non-zero length. The coordinates are integers between 0 and 10^6.

Output

For each test case: If a polyline does not intersect with itself only on its first and last point, print "INVALID POLYGON". Else, if two polygons touch or intersect, print "INTERSECTING POLYGONS". Else, if polygons are nested too deeply, print "INVALID NESTING". Lastly, if the description is correct, print "CORRECT".

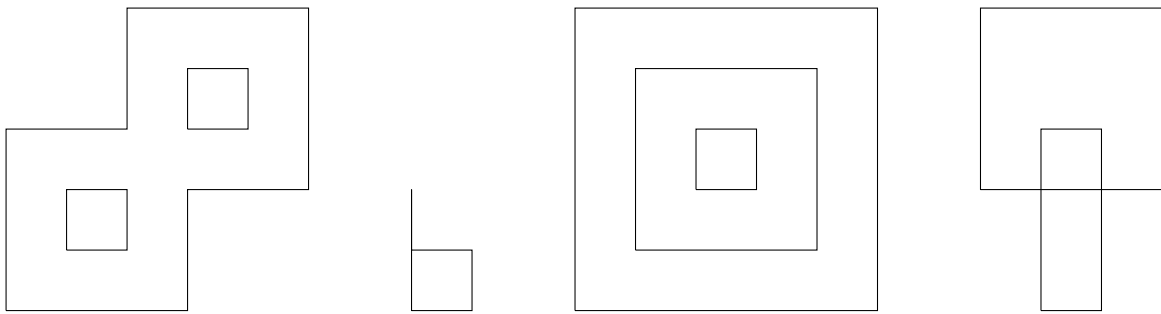*(Sample Input and Output can be found on the next page.)*

Figure 2 − Drawings of the four sample test cases.

Sample Input	Sample Output
4	CORRECT
3	INVALID POLYGON
9	INVALID NESTING
0 0 0 3 2 3 2 5 5 5 5 2 3 2 3 0 0 0	INTERSECTING POLYGONS
5	
1 1 1 2 2 2 2 1 1 1	
5	
3 3 3 4 4 4 4 3 3 3	
1	
6	
0 0 0 2 0 1 1 1 1 0 0 0	
3	
5	
2 2 2 3 3 3 3 2 2 2	
5	
1 1 1 4 4 4 4 1 1 1	
5	
0 0 0 5 5 5 5 0 0 0	
2	
5	
1 0 2 0 2 3 1 3 1 0	
5	
0 2 3 2 3 5 0 5 0 2	

Problem J

Not a subsequence

In this problem we consider strings over a fixed finite alphabet of size k. The alphabet contains the first k characters from the list

$$a, b, c, \ldots, z, A, B, C, \ldots, Z, 0, 1, \ldots, 9.$$

For every test case, we are given the value of k (notice that it cannot exceed 62), and consider only strings consisting of the first k characters from the list.

Given a string $s[1..n]$, we are interested in strings which are **not** its subsequences. Formally, a string $t[1..m]$ is a subsequence of a string $s[1..n]$ when one can choose **not necessarily contiguous** indices $1 \leq i_1 < i_2 < \ldots i_m \leq n$ such that $t[1] = s[i_1]$, $t[2] = s[i_2]$, ..., $t[m] = t[i_m]$. For example, acb is a subsequence of $babcaab$. Now, given a string $s[1..n]$, we would like to compute the smallest m such that there is a string $t[1..m]$, which is **not** a subsequence of $s[1..n]$. Additionally, we would like to count the number of such shortest strings $t[1..m]$. As the latter number can be quite large, output it modulo $10^9 + 7$.

Input

The input starts with the number of test cases $T \leq 100$. Then the descriptions of T test cases follow. A single test case consists of a single line containing the size of the alphabet k ($k \in [1, 62]$) and the string $s[1..n]$ ($n \in [1, 10^6]$). The string consists of the first k characters from a–$z$$A$–$Z$$0$–$9$.

Output

For every test case output one line containing two numbers. The first number is the smallest m such that there is a string $t[1..m]$ consisting of the first k characters from a–$z$$A$–$Z$$0$–$9$, which is **not** a subsequence of $s[1..n]$. The second number is the total count of such shortest strings $t[1..m]$ modulo $10^9 + 7$.

Sample Input	Sample Output
3	3 5
2 abba	1 52
62 0123456789	4 7
3 aabbcbbcbabcbab	

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Problem K
Pizza voting

You are training for a programming contest with your team members Alice and Bob. After some hours of hard training you want to have a break and eat pizza. You decided to order a big pizza for all three of you. But you have to choose the kind of pizza you want to eat.

You know your favorite kind. But Alice and Bob have other constraints: Alice is on a diet so she wants a pizza with less calories as possible. Bob is just mean to Alice so he votes for as much calories as possible.

You decide to vote on which kind of pizza you order. As voting for one pizza wouldn't lead anywhere, you decide to use a veto voting. So everyone of you veto on pizza in a round robin manner. First Alice vetos one pizza, then Bob vetos one, at last you are allowed to veto. Then Alice has the next veto again, then Bob etc. until only one pizza is left.

Reminder: Alice will always veto the pizza with the most calories. Bob vetos always the pizza with least calories. You try to be clever in such a way that your favorite pizza is the remaining pizza.

Input

The input starts with the number of pizzas n ($1 \leq n \leq 100\,000$) and the index of your favorite pizza p ($1 \leq p \leq n$) (1-indexed). Then follow the description of the n pizzas, each given in one line. The description consists of one integer c ($0 \leq c \leq 1\,000\,000$) giving the calories followed by a single word w giving the name of the pizza (up to 100 characters). The pizzas are ordered from low calories to high calories and the number of calories is unique for every pizza.

Output

Output one line. "YES" if you can vote in a way, such that your pizza will be selected. "NO" if you are not able to influence the vote in a way that your pizza will be selected.

Sample Input I	Sample Output I
5 2	YES
500 Margherita	
600 Salami	
700 Hawai	
800 Speciale	
900 Doener	

Sample Input II	Sample Output II
5 4	NO
500 Margherita	
600 Salami	
700 Hawai	
800 Speciale	
900 Doener	

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