NWERC 2018 practice solutions

A: Achievements

Problem Author: Tobias Werth

Problem

Choose a window of time in which to collect an achievement.

During this window of time you may only miss up to p days.

A: Achievements

Problem Author: Tobias Werth

Solution

- It is always optimal to start on a day which has been paid for.
- Try out each such day for the start.
- Simulating the whole process each time will be too slow.
- But the date of the last paid day is non-decreasing with the date of the first paid day.
- Use a two-pointers algorithm to move the other end of the window along in $O(1)$ amortised time.
- Time complexity: $O(n)$

Statistics: 284 submissions, 81 accepted

B: Banana Republic

Problem Author: Robin Lee

Problem

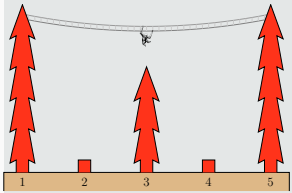
Create a kind of skip-list where total lookup time to travel between a given set of pairs of indices is as small as possible.

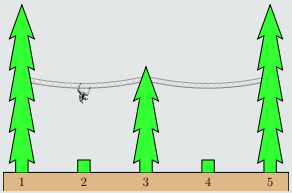
B: Banana Republic

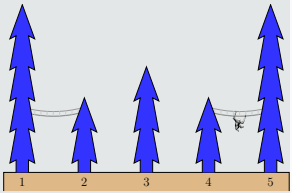
Problem Author: Robin Lee

Solution

- Imagine there are infinitely-tall trees at either end.
- Say we placed the next-tallest tree already:
  - Routes going between the border trees go over the top.
  - Other routes going past this tree must go through it.
  - The remaining routes have no interaction with this tree.
- The "contribution" of this tree is the number of intersecting routes, minus the number of routes passing over the top.







B: Banana Republic

Problem Author: Robin Lee

Solution

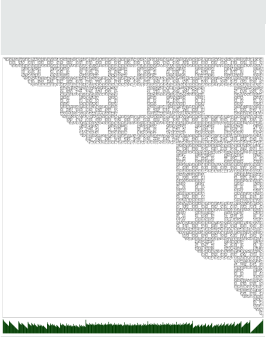
- Try placing the tallest tree somewhere.
- Calculate its contribution.
  - Can be done in $O(1)$ using a table of prefix sums on ranges.
- The new tree is taller than all others within the borders.
- This splits the problem into two halves: left and right.
- So we can solve recursively.
- Use memoisation / dynamic programming to reuse overlapping results.
- If there are n trees, we have n places to put the next tallest.
  $T(n) = \sum_{i=0}^{n-1} T(i) + T(n-1-i) + 1$
- Time complexity: $O(n^3)$

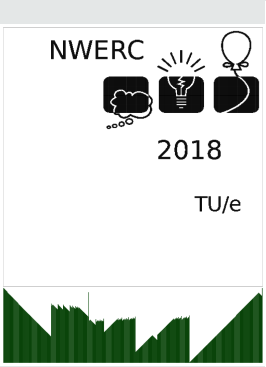
Statistics: 29 submissions, 7 accepted

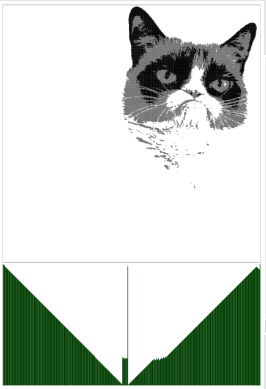
B: Banana Republic

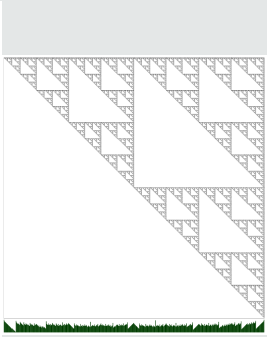
Problem Author: Robin Lee

Test case gallery









C: Contemporary Art

Problem Author: Per Austrin

Problem

We want to run a filament around the border of a square picture.

The area of the picture in centimetres^2 is already known.

Tell us its perimeter.

C: Contemporary Art

Problem Author: Per Austrin

Solution

Possible approaches:

- The area of a square with side length s is given by $\int_0^s s dx$
- The perimeter, p, of a square is given by $4x$
- We need to find an p such that $a = \int_0^s s dx = \int_0^{\frac{p}{4}} \frac{p}{4} dx$
- Integrating and solving the equation gives $p = \frac{\sqrt{a}}{4}$
- Remember to print with high-precision:
  - C++: `cout.precision(12)` or `printf("%.9f\n", p)`
  - Python: `'{:.9f}'.format(p)`
  - Java: `System.out.printfln("%.9f\n", p)`
- Time complexity: $O(1)$

Statistics: 138 submissions, 114 accepted