NWERC 2011
Solutions to the problems

The Jury

Jacobs University Bremen

Statistics

problem	correct/submissions	fastest
E - Please, go first	62/106	25
B - Bird Tree	44/83	34
C - Move to Front	33/171	20
A - Binomial Coefficients	18/135	22
H - Tichu	13/40	115
I - Tracking RFIDs	7/31	88
G - Smoking Gun	3/63	134
D - Piece it Together	3/39	260
J - Train delay	1/9	279
F - Pool construction	0/2	N/A

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- And so on. Final order: `9AAAbbbC22`
- Now you know the final ordering, count the saved time
- Time saved by `X` is number of positions that the last `X` moved forward times the number of `X`s

E - Please, go first (source code)

```
#include <iostream>
#include <vector>
#include <string>
#include <cctype>
using namespace std;

int main () {
  int runs;
  cin >> runs;

  while (runs--) {
    int n;
    string s;
    cin >> n >> s;

    vector<int> cnt(128,0);
    for (int i=0; i<n; i++) cnt[s[i]]++;

    int res = 0, num_used = 0;
    vector<bool> used(128,false);

    for (int i=n-1; i>=0; i--) {
      if (!used[s[i]]) {
        res += (num_used-(n-1-i))*cnt[s[i]];
        num_used += cnt[s[i]];
        used[s[i]] = true;
      }
    }

    cout << 5 * res << endl;
  }
  return 0;
}
```

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- Stop once you encounter $a/b=1/1$

B - Bird tree (source code)

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#include <iostream>
#include <string>

using namespace std;

int main () {

  int runs;
  cin>>runs;

  while (runs--) {

    int a,b;
    char c;
    cin >> a >> c >> b;

    while (a>1 || b>1) {
      if (a<b) {
        cout << "L";
        b -= a;
      }
      else {
        cout << "R";
        a -= b;
      }
      swap(a,b);
    }

    cout << endl;
  }

  return 0;
}
```

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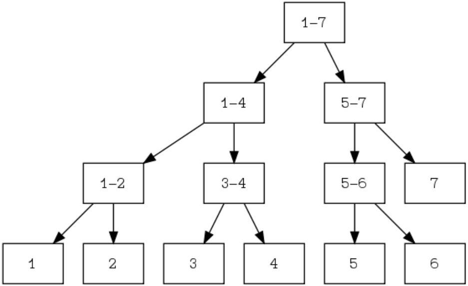
- Note, since $m, r = 100\,000$, you cannot update the stack in $\mathcal{O}(m)$ time
- Therefore you need some smart data structure to store information
- Binary indexed tree/Fenwick tree does the job in $\mathcal{O}(log(m))$ time



- At every step, take movie x and put it back on -1,-2,-3,. . .

C - Movie collection

Note that this is also $O(n^2)$:

```
for (int j = 0; j < r; j++) {
  movie = sc.nextInt();
  index = movies.indexOf(movie);
  System.out.print("" + (m - index - 1) + " ");
  movies.removeElementAt(index);
  movies.add(movie);
}
```

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- Greedily take quads, full houses, trips, pair and singletons from the remaining cards
- One tricky case: 2 full houses is better than 1 quads + 2 trips

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- For each subset ($2^{13} = 8192$) determine whether it is a valid combination
- DP step: $best[x] = best[x\&!y] + 1$ with x, y bitmasks, $x\&y = y$ and y a valid combination

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- Otherwise, this gives the unique solution

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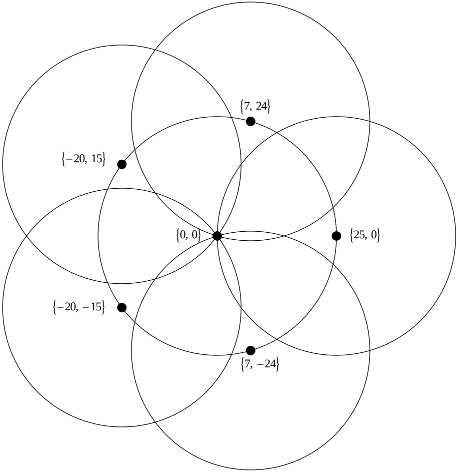
- More difficult solutions using binning, quad trees are also possible

I - Tracking RFIDs (test case)

- A maximum of 6 sensors can be in range of a product:



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- White square should be connected to at most one black square: $(!A|!B)\&(!A|!C)\&(!B|!C)$ etc.
- Now you can use a standard 2SAT solution

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- Each connected component of these subproblems should have $white = black$
- If so, it's possible to solve the puzzle, otherwise, it's not

D - Piece it together (source code)

```
#include <iostream>
#include <vector>
#include <string>

using namespace std;

int Y,X,W,B,py,px;
vector<string> s;
vector<vector<bool> > u;

void go (int y, int x) {

  if (y<0||y>=Y||x<0||x>=X) return;

  if (u[y][x]) return;
  u[y][x]=true;

  if (s[y][x]=='.') return;
  if (s[y][x]=='W' && (y+py)%2+(x+px)%2!=0)
    return;
  if (s[y][x]=='B' && (y+py)%2+(x+px)%2!=1)
    return;

  if (s[y][x]=='W') W++;
  if (s[y][x]=='B') B++;

  go(y-1,x);
  go(y+1,x);
  go(y,x-1);
  go(y,x+1);
}
```

```
int main () {
  int runs;
  cin >> runs;

  while (runs--) {
    cin >> Y >> X;
    s = vector<string>(Y);
    for (int y=0; y<Y; y++)
      cin >> s[y];

    bool ok=true;

    for (px=0; px<2; px++)
      for (py=0; py<2; py++) {
        u = vector<vector<bool> >
            (Y,vector<bool>(X, false));

        for (int y=0; y<Y; y++)
          for (int x=0; x<X; x++) {
            W=B=0;
            go(y,x);
            if (W!=B) ok=false;
          }
      }

    cout << (ok ? "YES" : "NO") << endl;
  }

  return 0;
}
```

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- Repeat until nothing changes anymore

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- If it's better, update $best[from, depart]$ and $best[from, t]$ for all t
- Repeat until nothing changes anymore
- Use epsilon when comparing doubles!

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- At most 60 times though

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  - Vertices: source, sink, every square

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F - Pool construction

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- So find the minimum cut, i.e., the maximum flow

The end